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CONDITIONAL BEST LINEAR INVARIANT
ESTIMATION OF THE LOCATION AND SCALE
PARAMETERS OF THE CAUCHY DISTRIBUTION
BY THE USE OF ORDER STATISTICS

THESIS

GAM/~~MATH~~ MA/72-3

Ralph M. Spory, Jr.
Captain USAF

1972

CONDITIONAL BEST LINEAR INVARIANT ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION BY THE USE OF ORDER STATISTICS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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Graduate Aerospace-Mechanical Engineering

March 1972

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Preface

This thesis is a continuation and extension of previous work by graduate students at the Air Force Institute of Technology in the area of parameter estimation using order statistics of samples from a given distribution. The tables of linear coefficients developed in this report will enable the user to obtain the best linear invariant estimate of the location and scale parameters of the Cauchy distribution for sample sizes of N=5(1)20 very efficiently. An attempt was made in the report to provide a clear development of the theory by which these linear coefficients are obtained. In addition, the Fortran program required to calculate and table the linear coefficients is included in Appendix C. The subroutine used to solve the matrix equations is a modification of the Matrix Equation Solver Fortran Subroutine from the Computer Science Center, Wright-Patterson AFB, Ohio.

I also wish to acknowledge my debt to Professor Albert H. Moore, my advisor, for proposing this area of study and for his assistance and encouragement.

Ralph M. Spory, Jr.

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Abstract

Linear coefficients which can be applied to sample data from a Cauchy distribution to obtain estimates of the location and scale parameters are developed and tabled. Several previous works have presented such tables for nearly best linear unbiased estimation and best linear unbiased estimation of the parameters. The estimates developed in this paper are best in the sense that they possess minimum mean square error. By using exact values of the means, variances, and covariances of the Cauchy standardized order statistics and minimizing the mean square error function, matrix equations are developed and solved to obtain the required coefficients. These coefficients and values of the MSE are tabled for minimally censored sample sizes of 5 to 20 and for samples which have been additionally censored from above and symmetrically. Procedures for using the tables and several illustrative calculations demonstrate the simplicity of this estimation technique. The Fortran programs required to calculate and table the above values are included in Appendix C.

CONDITIONAL BEST LINEAR INVARIANT ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION BY THE USE OF ORDER STATISTICS

I. Introduction

Statement of the Problem

Objective. The objective of this thesis is to develop a table of linear coefficients which can be easily applied to sample data from a Cauchy distribution to determine estimates of the location and scale parameters of the distribution. These estimates of the parameters are conditional best linear invariant estimates. These terms and the properties of the estimators are defined in the next section.

Definition of Terms. The Cauchy distribution is a continuous distribution which is frequently introduced to students as an example of a distribution for which the moments do not exist (Ref 6:134). The cumulative distribution function (cdf) is given by

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \text{ARCTAN } \frac{x-t}{s} \quad , \quad -\infty<x<\infty \tag{1}$$

and the probability density function (pdf) is given by

$$f(x) = \frac{s}{\pi[s^2+(x-t)^2]} \quad , \quad -\infty<x<\infty \tag{2}$$

where s is the scale parameter and t is the location parameter.

Conditional estimation of a parameter is defined as estimation of an unknown parameter when the second parameter is known. In this case the location parameter is estimated conditioned on the value of the scale parameter, and the scale parameter is estimated conditioned on the value of the location parameter.

The best linear invariant estimator is best in the sense that it is a minimum mean-square-error estimator. The mean square error of the estimator is given by

$$MSE = E[(s^*-s)^2] \tag{3}$$

where s is the true value of the parameter and s* is the estimated value of the parameter.

For conditional estimation the mean square error is given by

$$MSE = E[(s^*|t-s)^2] \tag{4}$$

where $s^*|t$ is the estimator of the parameter conditioned on the value of t, and t is the value of the second parameter.

Linear estimation is a technique based on the theory of order statistics, where each ordered sample value is assigned a weight, or linear coefficient, and the coefficients are calculated so as to obtain the best estimate of the parameter. In this case the best estimate is the invariant estimate as defined above. If one uses this method, the estimate of the parameter is given by

$$s^* = \sum_{i=1}^{n} a_i X_i \qquad (5)$$

where the a_i's are the linear coefficients, the X_i's are the ordered sample values or the order statistics, and n is the sample size.

The conditional estimator is then given by

$$s^*|t = \sum_{i=1}^{n} a_i X_i - At \qquad (6)$$

where t is the known parameter and A is a constant that is determined by the calculation. The significance of this constant will be explained in Chapter IV.

Order statistic theory, the Cauchy distribution, and the method used to solve for the linear coefficients are developed further in the remaining chapters of this paper.

Significance. When the location and scale parameters of the Cauchy distribution are known, the function is completely defined, and it can be used in a decision-making process where one is working with data which is distributed according to the Cauchy law. The tables of coefficients developed in this thesis will allow the user to estimate the values of these unknown parameters. These estimates can be obtained very efficiently with only the use of a desk calculator. The ordered sample values are simply multiplied by the appropriate coefficients and the results summed to calculate the estimate.

This method of estimation provides a great savings in time in the case of the Cauchy distribution, as the traditional methods of obtaining these estimates either do not apply to the distribution or are very time-consuming. Two of these methods are the method of moments (Ref 11:186) and the method of maximum likelihood (Ref 11:170). The method of moments cannot be applied, since the moments of the Cauchy distribution do not exist, and the maximum likelihood estimate, which is convenient to obtain for some other distributions, requires a great amount of computational effort. Barnett (Ref 1) points out that the frequent occurrence of multiple zeros of the derivatives of the logarithm of the likelihood function requires a complete scan of the likelihood function to locate the maximum which corresponds to the maximum likelihood estimate. The tables of linear coefficients in this thesis will provide the user with an estimate of these parameters for sample sizes of 5 to 20.

Background Information

Work on parameter estimation based on order statistic theory has been carried out by the students at the Air Force Institute of Technology, under the direction of Professor Albert H. Moore, and sponsorship of Dr. H. Leon Harter (ARL, Wright-Patterson AFB), since 1963. These works include parameter estimation of the Cauchy, Weibull, normal, log-normal, logistic, and extreme value distributions. Parameter estimation of the Cauchy distribution includes the works of Chamberlain (Ref 2), Jonson (Ref 9) and Stark (Ref 16).

Chamberlain developed and tabled the coefficients for nearly best linear unbiased estimation of the location and scale parameters for sample sizes 15(1)40. Jonson computed the coefficients for conditional best linear unbiased estimation of the parameters of the Cauchy distribution and compared the efficiency of these estimators with the efficiency of Stark's best linear unbiased estimators. The estimators developed by Jonson and Chamberlain are called nearly best estimators because the approximate values of the covariances of the order statistics given by Blom's approximation were used instead of the exact covariances of the order statistics. Stark's work developed linear coefficients for simultaneous estimation of the location and scale parameters by using the exact values of the means, variances, and covariances of the standardized order statistics.

The works of Chamberlain and Jonson are based largely on the methods presented by Barnett (Ref 1:1205). Barnett tabled the coefficients for the best linear unbiased estimate of the location parameter of the distribution for sample sizes of 5 to 20. The exact values of the means, variances, and covariances of the Cauchy order statistics were calculated to four decimal-place accuracy. The values of the covariances were obtained by numerical integration of expressions containing the joint pdf of the order statistics (see Chapter IV). These functions were integrated over the relevant triangular region by a two-dimensional

extension of the composite Simpson procedure. Although this computation required a large number of steps to obtain the desired accuracy, it did prove feasible.

The median of the sample data has traditionally been used as an estimate of the location parameter of the Cauchy distribution. Cramer (Ref 4:708) states that the variance of this estimator is $\pi^2/4n$ for large samples. Rider (Ref 13:322) shows that this is not a very accurate estimate of the variance of the median for small sample sizes. Rider has tabled the actual variances of the median for small sample sizes.

In 1964, Rothenberg et al. proposed a class of estimators of the location parameter of the Cauchy distribution which is the arithmetic average of a central subset of the sample order statistics. The sample median is a member of this subset, but it was shown that the average of the middle quarter of the ordered samples has a lower asymptotic variance than does the median.

In 1970, Chan (Ref 3:851) proposed a conditional asymptotically best linear estimator of the location and scale parameters based on a few of the order statistics. He has tabled coefficients for K=1(1)10 where K is the number of order statistics selected from a large sample. These estimators yield more than 92 percent asymptotic relative efficiency, in the Cramer-Rao sense, for $K \geq 4$.

Report Organization

In order to develop the linear coefficients for the conditional best linear invariant estimation of the parameters of the Cauchy distribution, the distribution and methods by which it is generated will be discussed in Chapter II. Chapters III and IV present order statistic theory and the linear estimation procedure. Chapter V describes the tables of linear coefficients and gives examples of the calculations required to obtain the desired estimators. Tables of the values of the mean-square-error function and the linear coefficients are included in Appendices A and B. Appendix C contains the Fortran programs required to calculate and table the above values.

Assumptions

There are two assumptions made in this report. It is assumed that the sample data are known to come from a Cauchy distributed parent population and that the parameter not being estimated is known, in the case of conditional estimation. In the case of simultaneous estimation it is only assumed that the sample data are known to come from a Cauchy distributed parent population.

II. The Cauchy Distribution

Introduction

The Cauchy distribution is a continuous, symmetric distribution. The cdf and pdf are given by Equations (1) and (2). The plot of the pdf is similar to that of the more familiar normal distribution, except that the curve approaches the axis much more slowly and the tails are thicker. Figure 1, on the following page, is a plot of the density function for three different scale parameters. Feller (Ref 5:57) provides an excellent description of the Cauchy distribution, its peculiarities, and methods by which the distribution is generated.

Example of the Cauchy Distribution

A mirror is arranged parallel to an opposing wall at a distance S from the wall, and the mirror is free to rotate on a vertical axis at A which is located on a line perpendicular to the wall at 0. The angle ϕ is measured from this line to the perpendicular to the surface of the mirror. The mirror reflects a ray of light on the wall at a distance X from the point 0. Now, if the angle ϕ is chosen at random between $-\pi/2$ and $\pi/2$, the random variable, X, is Cauchy distributed and the density of the distribution is given by Equation (2) with a location parameter of zero (Ref 5:57).

The density of the random variable, X, can easily be verified by a method due to Meyer (Ref 10:88-89). In the above example

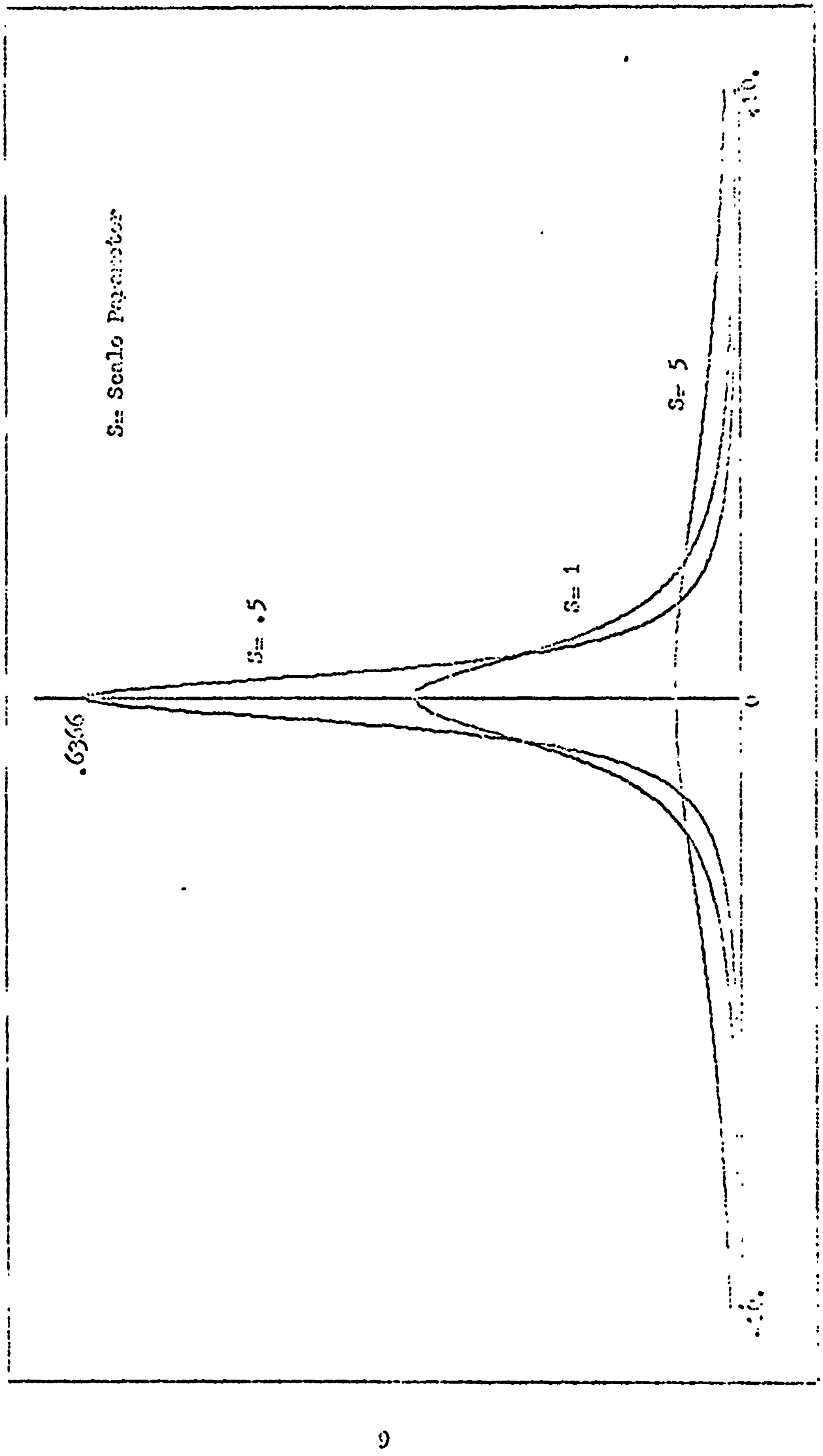

Fig. 1. The Cauchy Probability Density Function

$$f(\phi) = \frac{1}{\pi} \quad \text{for} \quad -\frac{\pi}{2}<\phi<\frac{\pi}{2}$$

is a uniform density. Then

$$g(x) = \frac{1}{\pi}\ \frac{d\phi}{dx} \tag{7}$$

where $\phi = \tan^{-1}(\frac{x}{s})$. Therefore

$$g(x) = \frac{1}{\pi}\ \frac{1}{1+\frac{x^2}{s^2}}\ \frac{1}{s} \tag{8}$$

$$g(x) = \frac{1}{\pi}\ \frac{s}{[s^2+x^2]} \quad \text{for } -\infty<x<\infty \tag{9}$$

This is the Cauchy pdf with scale parameter s and location parameter zero.

Generation of the Cauchy Density

The Cauchy density may be generated in many ways. The Student's t density with n=1 is a Cauchy density. If x and y are two independent random variables from a standardized normal distribution, the quotient of these random variables is a standardized Cauchy distribution. In addition, if the random variable x is Cauchy distributed with a scale parameter of 1 then 1/x has the same density. Once again, these densities may be verified by the method described by Meyer (Ref 5:109-110).

Properties of the Cauchy Distribution

Due to the thick tails of the Cauchy distribution, estimation of its center is very difficult. The moments of the

Cauchy distribution do not exist. Jonson (Ref 9:11) shows the characteristic function of the Cauchy random variable to be

$$C_X(t) = \exp(ict-b|t|) \tag{10}$$

where C is the location parameter and b is the scale parameter, and that the moments do not exist since the partial derivatives of $C_X(t)/i^k$ with respect to t evaluated at t=0 are infinite.

It can also be shown directly, that the first moment about the origin of the Cauchy distribution does not exist (Ref 6:145).

III. Order Statistic Theory

Introduction

To calculate the conditional best linear invariant estimates of the parameters of the Cauchy distribution the exact values of the means, variances, and covariances of the Cauchy order statistics are required. The order statistic theory and application of this theory to the Cauchy distribution will be reviewed in this chapter. Reference 15 is a rather complete collection of contributions to order statistic theory and includes articles up to 1962. The following development follows that of Chapter II from the above reference.

Order Statistics

Definition. If a random sample of size n (x_1, x_2, ... x_n) is taken from a population, these independent random variables can be rearranged so that

$$x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$$

When the variables are arranged in order of magnitude, they are called order statistics of the sample. Since these samples are from a continuous population

$$P(x_{(i)} = x_{(j)}) = 0 \quad \text{for all } i \neq j$$

Density. The pdf of the ith order statistic, $x_{(i)}$, is given by

$$g(x_{(i)}) = \frac{n!}{(i-1)!(n-i)!}[F(x_{(i)})]^{i-1}[1-F(x_{(i)})]^{n-i}f(x_{(i)}) \tag{11}$$

where $F(x_{(i)})$ = cdf of x evaluated at $x=x_{(i)}$

$f(x_{(i)})$ = pdf of x evaluated at $x=x_{(i)}$ (Ref 15:12)

The joint distribution of the ith and jth order statistics ($i<j$) is given by

$$g(x_{(i)},x_{(j)}) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F(x_{(i)})]^{i-1}[F(x_{(j)}) - F(x_{(i)})]^{j-i-1} \cdot [1-F(x_{(j)})]^{n-j}f(x_{(i)})f(x_{(j)}) \tag{12}$$

for $x_{(i)}<x_{(j)}$ (Ref 15:12)

From Equations (11) and (12) expressions for the expected values and covariances of the Cauchy order statistic can be developed.

Expected Values. Let x be a continuous random variable with pdf f, then the expected value of x is given by

$$E[x] = \int_{-\infty}^{\infty} xf(x)dx \qquad \text{(Ref 10:121)} \tag{13}$$

Using Equations (13) and (14), one finds that the expected value of the ith order statistic is given by

$$E[x_{(i)}] = \int_{-\infty}^{\infty} x_{(i)}g(x_{(i)})dx_{(i)} \tag{14}$$

And the expected value of the product of the ith and jth order statistics is given by

$$E[x_{(i)}x_{(j)}] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_{(i)}x_{(j)}g(x_{(i)}x_{(j)})dx_{(i)}dx_{(j)} \qquad (15)$$

where $g(x_{(i)},x_{(j)})$ is as defined by Equation (12).

Variance and Covariance. A two-dimensional random variable $(x_{(i)},x_{(j)})$ possesses a property called the covariance of $x_{(i)},x_{(j)}$. In this case the random variables are order statistics, and in a sense the covariance is a measure of the dependence between the two values of the order statistics. The covariance of $(x_{(i)},x_{(j)})$ is formally defined as the product moment about the respective expected values of the order statistics. Meyer (Ref 10:144) defines the covariance of the two random variables as follows:

$$\mathrm{Cov}(x_{(i)},x_{(j)}) = E\{[x_{(i)}-E(x_{(i)})][x_{(j)}-E(x_{(j)})]\} \qquad (16)$$

It can be easily shown from (16) that:

$$\mathrm{Cov}(x_{(i)},x_{(j)}) = E[x_{(i)}x_{(j)}]-E[x_{(i)}]\cdot E[x_{(j)}] \qquad (17)$$

The variance of the ith order statistic may be considered as a special case (i=j) of Equation (16) where the Var $[x_{(i)}]$ is given by

$$\mathrm{Var}[x_{(i)}] = E\{[x_{(i)}-E(x_{(i)}]^2\} \qquad (18)$$

With the expressions presented in this chapter, the expected values, variances, and covariances of the Cauchy order statistics can be developed.

Standardized, Cauchy Order Statistics. Let $x_{(i)}$, $x_{(2)}$, $\ldots x_{(n)}$ be a set of ordered sample values from a Cauchy

distributed parent population. A new set of standardized order statistics can be developed where

$$U_{(i)} = \frac{x_{(i)}-t}{s} \qquad -\infty<U_{(i)}<\infty \tag{19}$$

with $\qquad U_{(1)} <\ldots<U_{(i)} <\ldots<U_{(n)}$

If the pdf of x is $f(x) = \dfrac{s}{\pi[s^2+(x-t)^2]}$ (2)

then the density of U is given by

$$p(u) = \frac{1}{\pi[1+u^2]} \qquad -\infty<U<\infty \tag{20}$$

and its cdf is given by $P(u) = \frac{1}{2} + \frac{1}{\pi}$ ARCTAN u (21)

The pdf of the ith standardized order statistic from Equation (11) is given by

$$q(u_{(i)}) = \frac{n!}{(i-1)!(n-i)!}[P(u_{(i)}]^{i-1}[1-P(u_{(i)}]^{n-i}p(u_{(i)}) \tag{22}$$

and the joint pdf of the standardized order statistics is given by Equation (12) with x replaced by u, g by p, F by P and f by p.

The above expression can be simplified by making the following substitution:

Let $\theta_{(i)}$ = ARCTAN $u_{(i)}$, $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, $-\infty<u_{(i)}<\infty$ (23)

where $du_{(i)} = (1 + \tan^2\theta_{(i)})d\theta_i$ (24)

and $\phi_{(j)}$ = ARCTAN $u_{(j)}$ $\qquad -\frac{\pi}{2}<\phi<\frac{\pi}{2}$ (25)

then from Equation (22)

$$q(u_{(i)}) = \frac{n!}{(i-1)!(n-i)!\pi^n}[\theta + \frac{\pi}{2}]^{i-1}[\frac{\pi}{2}-\theta]^{n-i}\frac{1}{(1+\tan^2\theta)} \qquad (26)$$

and

$$q(u_{(i)}u_{(j)}) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!\pi^n}[\theta+\frac{\pi}{2}]^{n-i}$$

$$[(\theta+\frac{\pi}{2}) - (\emptyset+\frac{\pi}{2})]^{j-i-1} \cdot [\frac{\pi}{2}-\emptyset]^{n-j}\frac{1}{(1+\tan^2\theta)}\frac{1}{(1+\tan^2\emptyset)} \qquad (27)$$

By substituting these equations into Equations (14), (15), (17), and (18) the desired expressions for the expected values, variances and covariances of the standardized order statistics are obtained.

$$E[u_{(i)}] = \frac{n!}{(i-1)!(n-i)!\pi^n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan\theta[\theta+\frac{\pi}{2}]^{i-1}[\frac{\pi}{2}-\theta]^{n-i}d\theta_i \qquad (28)$$

$$\mathrm{Cov}[u_{(i)},u_{(j)}] = \frac{n!}{(i-1)!(j-i-1)!(n-j)!\pi^n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\emptyset}\tan\emptyset\tan\theta[\theta+\frac{\pi}{2}]^{i-1}$$

$$\cdot[(\theta+\frac{\pi}{2})-(\emptyset+\frac{\pi}{2})]^{j-i-1}[\frac{\pi}{2}-\emptyset]^{n-j}d\theta d\emptyset - \frac{(n!)^2}{(i-1)!(n-i)!(j-1)!(n-j)!\pi^n}$$

$$\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan\theta[\theta+\frac{\pi}{2}]^{i-1}[\frac{\pi}{2}-\theta]^{n-i}d\theta\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan\emptyset[\emptyset+\frac{\pi}{2}]^{j-1}[\frac{\pi}{2}-\emptyset]^{n-j}d\emptyset, i<j \qquad (29)$$

$$\mathrm{Var}[u_{(i)}] = \frac{n!}{(i-1)!(n-j)!\pi^n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan^2\theta_{(i)}[\theta_{(i)}+\frac{\pi}{2}]^{i-1}[\frac{\pi}{2}-\theta_{(1)}]^{n-j}d\theta_{(i)} \qquad (30)$$

Solution of the Expected Value and Covariance Equations

Equations (28), (29), and (30) cannot be evaluated in terms of elementary functions, but they can be evaluated by numerical integration techniques with the aid of an electronic computer. The integral factor of Equation (28) does not converge for values of $i=1$ or n, and Equations (29) and (30) do not converge for values of $i=1, 2, n$, and $n-1$. Barnett (Ref 1:1209) points out that for these values of i, the $\cos\theta$ and $\cos\phi$ factors in the denominator of the respective integrands are dominant as θ and ϕ tend toward their limiting values, and it is therefore apparent that the means of the first and last order statistics do not exist, and the variances and covariances of the first two and last two order statistics do not exist.

The task of evaluating the remaining quantities is somewhat reduced due to the symmetry of the Cauchy distribution. For the expected values

$$E[u_{(i)}] = -E[u_{(n+1-i)}] \quad \text{for even } n,\ i \neq 1 \text{ or } n \qquad (31)$$

$$E[u_{(n+1)/2}] = 0 \quad \text{for odd } n \qquad (32)$$

and for the covariances of the standardized order statistics:

$$\begin{aligned} \mathrm{Cov}[u_{(i,j)}] &= \mathrm{Cov}[u_{(n+2-i,n+1-j)}] = \mathrm{Cov}[u_{(j,i)}] \\ &= \mathrm{Cov}[u_{(n+2-j,n+2-i)}] \\ &\quad \text{for } i<j \text{ and } i \neq 1,2,n, \text{ and } n-1 \qquad (33) \end{aligned}$$

Even with this reduction in the number of quantities to be computed, the integrals must be approximated by an electronic computer. Barnett (Ref 1) calculated these expected values for n=3(1)20 and the variances and covariances for n=5(1)20 to four-decimal-place accuracy. Although the expected values and variances can be calculated quite efficiently, the double integration in the covariance expression requires extensive computer time. Stark (Ref 16:44-48) calculated and tabled these quantities for sample sizes of 5(1)20 to six-decimal-place accuracy. The expected values and covariances calculated by Stark were read into the main programs in Appendix C for the calculation of the linear coefficients for conditional best linear invariant estimation of the parameters.

IV. Linear Parameter Estimation

Introduction

The purpose of this thesis is to develop the coefficients required to obtain best linear invariant estimates of the parameters of the Cauchy distribution. This chapter explains linear estimation, develops the estimator with the minimum mean square error for both parameters, and describes the method used to solve the resulting simultaneous equations. The coefficients are calculated for the minimally censored sample, which consists of the ordered sample less the four extreme order statistics (the first two and the last two), and for samples which are additionally censored from above and symmetrically. In addition the values of the mean square error function are computed. All of these quantities are tables in the Appendices.

Linear Estimation

Linear estimation is a form of estimation where the ordered data are assigned weights and the new values are summed to give the estimate of the desired parameter. These weights are the coefficients and they are calculated so as to obtain the best linear estimator of the parameter. In this case the best estimator is the one with the minimum mean square error. The expressions required to obtain these best estimators are developed in the next section.

<u>Mean Square Error</u>

<u>Location Parameter</u>. From Equation (6) the conditional estimate of the location parameter is given by

$$t^*|S = \sum_{i=1}^{N} a_i x_i - AS \tag{34}$$

where t^* is the estimate of the location parameter

a_i is the coefficient to be computed

S is the scale parameter

A is a constant to be computed

N is the sample size

and from Equation (4) the mean square error of the estimator of the location parameter is given by

$$MSEL = E[(t^*|S-t)^2] \tag{35}$$

where t is the true value of the location parameter. MSEL is the quantity to be minimized.

From Equation (35)

$$MSEL = E[(t^*|S)^2 - 2t(t^*|S) + t^2] \tag{36}$$

From Ref 10:134

$$Var[x] = E[x^2] - (E[x])^2 \tag{37}$$

therefore $$Var[t^*|S] = E[(t^*|S)^2] - (E[t^*|S])^2 \tag{38}$$

and $$MSEL = Var[t^*|S] + (E[t^*|S] - t)^2 \tag{39}$$

Now by the substitution of Equation (34),

$$MSEL = Var[\sum_{i=1}^{N} a_i x_{(i)} - AS] + (E[\sum_{i=1}^{N} a_i x_{(i)} - AS] - t)^2 \tag{40}$$

Freund (Ref 6:173-174) shows that for a given set of random variables $x_1, x_2, \ldots x_n$ where $Y = \sum_{i=1}^{N} a_i x_i$, a linear combination of N random variables,

$$E[Y] = \sum_{i=1}^{N} a_i E[x_i] \tag{41}$$

and $$Var[Y] = \sum_{i=1}^{N} a_i^2 Var[x_{(i)}] + 2\sum_{i<j}\sum a_i a_j Cov[x_{(i)} x_{(j)}] \quad \text{for } i<j \tag{42}$$

Now using relations (41) and (42) Equation (40) becomes

$$MSEL = \sum_{i=1}^{N} a_i^2 Var[x_{(i)}] + 2\sum_{i<j}\sum a_i a_j Cov[x_{(i)} x_{(j)}] + (\sum_{i=1}^{N} a_i E[x_{(i)}] - AS - t)^2 \tag{43}$$

The expected values, variances and covariances developed in Chapter III were for standardized order statistics, where the standardized order statistic was defined by Equation (19). Therefore

$$x_{(i)} = SU_{(i)} + t \tag{44}$$

$$E[x_{(i)}] = SE[u_{(i)}] + t \tag{45}$$

$$Var[x_{(i)}] = s^2 Var[u_{(i)}] \tag{46}$$

$$Cov[x_{(i)} x_{(j)}] = s^2 Cov[u_{(i)} u_{(j)}] \tag{47}$$

Now define the following symbols:

Let $$\mu_i = E[u_{(i)}]$$

$$\sigma_i = Var[u_{(i)}]$$

$$\sigma_{ij} = Cov[u_{(i)}u_{(j)}]$$

Now by substituting these relations into Equation (43)

$$MSEL = \sum_{i=1}^{N} a_i^2 s^2 \sigma_{ii} + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} a_i a_j s^2 \sigma_{ij} + [\sum_{i=1}^{N} a_i(s\mu_i + t) - AS - t]^2 \quad (48)$$

and adding the constraint that $\sum_{i=1}^{N} a_i = 1$ and applying this equation to a minimally censored Cauchy sample of size N-4 (the two extreme order statistics are removed from each end), the following equation is obtained

$$MSEL = s^2(\sum_{i=3}^{N-2} a_i^2 \sigma_{ii} + 2 \sum_{i=3}^{N-2} \sum_{j=i+1}^{N-2} a_i a_j \sigma_{ij} + [\sum_{i=3}^{N-2} a_i \mu_i - A]^2) \quad (49)$$

Scale Parameter. An expression similar to Equation (49) can be developed for the scale parameter where the mean square error of the estimated scale parameter is given by

$$MSES = E[(S^*|t - S)^2] \quad (50)$$

where S is the true value of the scale parameter,

S* is the estimate of S, and

t is the true value of the location parameter.

Let $$S^*|t = \sum_{i=1}^{N} d_i x_{(i)} - Dt \quad (51)$$

and substitute into Equation (50) to obtain

$$MSES = Var[\sum_{i=1}^{N} d_i x_{(i)} - Dt] + (E[\sum_{i=1}^{N} d_i x_{(i)} - Dt] - S)^2 \quad (52)$$

And by the use of Equations (42), (45), (46), and (47), Equation (52) becomes

$$MSES = \sum_{i=1}^{N} d_i^2 s^2 \sigma_{ii} + 2\sum_{i=1}^{N} \sum_{j=i+1}^{N} s^2 d_i d_j \sigma_{ij} + \left[\sum_{i=1}^{N} d_i (s\mu_i + t) - Dt - S\right]^2 \quad (53)$$

Now by adding the constraint that $\sum_{i=1}^{N} d_i = D$ and considering a minimally censored Cauchy sample the mean square error of the estimate of the scale parameter is given by

$$MSES = s^2\left(\sum_{i=3}^{N-2} d_i^2 \sigma_{ii} + 2\sum_{i=3}^{N-2} \sum_{j=i+1}^{N-2} d_i d_j \sigma_{ij} + \left[\sum_{i=3}^{N-2} d_i \mu_i - 1\right]^2\right) \quad (54)$$

Equations (54) and (49) are the required expressions to compute the mean square error, but in this case, it is desired to minimize these quantities.

Minimization of the Mean Square Error

Both expressions for the mean square error contain the scale parameter squared. At this point, the problem is to determine the values of the a_i's, A, d_i's and D which minimize the respective MSE function, and these values will be the same if the functions are minimized without the (s^2) term.

Taylor (Ref 17:198) describes Lagrange's method of minimizing a function of several variables subject to a constraint. In applying this method the original function is modified by adding the constraint equation multiplied by a Lagrangian multiplier, and then taking partial derivatives of the function with respect to each variable and multiplier.

The resulting derivatives are set equal to zero to form a set of simultaneous equations. These equations are then solved for the values of the variables and multipliers which minimize the function.

Matrix Equation. To develop the matrix equations for the calculation of the required coefficients, Equation (49) is modified by adding the constraint, $\sum_{i=3}^{N-2} a_i=1$, to give

$$L = \sum_{i=3}^{N-2} a_i^2\sigma_{ii}+2\sum_{i=3}^{N-2}\sum_{j=i+1}^{N-2} a_i a_j \sigma_{ij}+[\sum_{1=3}^{N-2} a_i\mu_i-A]^2+\lambda(\Sigma a_i-1) \quad (55)$$

Now if N=7 is the total sample size and m=3 is the size of the sample after censoring, application of the Lagrangian method results in the following set of equations.

$$\frac{\partial L}{\partial \lambda} = a_3+a_4+a_5-1 = 0$$

$$\frac{\partial L}{\partial A} =-a_3\mu_3-a_4\mu_4-a_5\mu_5+A = 0$$

$$\frac{\partial L}{\partial a_3}= a_3(\sigma_{33}+\mu_3^2)+a_4(\sigma_{34}+\mu_3\mu_4)+a_5(\sigma_{35}+\mu_3\mu_5)-A\mu_3+\frac{\lambda}{2} = 0 \quad (56)$$

$$\frac{\partial L}{\partial a_4}= a_3(\sigma_{43}+\mu_4\mu_3)+a_4(\sigma_{44}+\mu_4^2)+a_5(\sigma_{45}+\mu_4\mu_5)-A\mu_4+\frac{\lambda}{2} = 0$$

$$\frac{\partial L}{\partial a_5}= a_3(\sigma_{53}+\mu_5\mu_3)+a_4(\sigma_{54}+\mu_5\mu_4)+a_5(\sigma_{55}+\mu_5^2)-A\mu_5+\frac{\lambda}{2} = 0$$

The above equations in matrix form are:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -\mu_3 & -\mu_4 & -\mu_5 \\ 1 & -\mu_3 & (\sigma_{33}+\mu_3^2) & (\sigma_{34}+\mu_3\mu_4) & (\sigma_{35}+\mu_3\mu_5) \\ 1 & -\mu_4 & (\sigma_{43}+\mu_4\mu_3) & (\sigma_{44}+\mu_4^2) & (\sigma_{45}+\mu_4\mu_5) \\ 1 & -\mu_5 & (\sigma_{53}+\mu_5\mu_3) & (\sigma_{54}+\mu_5\mu_4) & (\sigma_{55}+\mu_5^2) \end{bmatrix} \begin{bmatrix} \frac{\lambda}{2} \\ A \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (57)$$

A similar procedure for Equation (54) results in the following matrix equation for the coefficients of the scale parameter.

$$\begin{bmatrix} (\sigma_{33}+\mu_3^2) & (\sigma_{34}+\mu_3\mu_4) & (\sigma_{35}+\mu_3\mu_5) \\ (\sigma_{43}+\mu_4\mu_3) & (\sigma_{44}+\mu_4^2) & (\sigma_{45}+\mu_4\mu_5) \\ (\sigma_{53}+\mu_5\mu_3) & (\sigma_{54}+\mu_5\mu_4) & (\sigma_{55}+\mu_5^2) \end{bmatrix} \begin{bmatrix} d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} \quad (58)$$

where $D = d_3 + d_4 + d_5$

The matrix Equations (57) and (58) include matrices of the expected values and covariance of standardized order statistics and the column vectors of the desired variables. These equations can be solved for the column vector of m + 2 variables for the location parameter and m variables for the scale parameter.

Solution of the Matrix Equations. The above matrix equations were solved for sample sizes of N = 5(1)20 on the CDC 6600 computer. Basically, the main Fortran program reads in the values of the means, variances, and covariances, and

the matrix equations are calculated and then solved by a subroutine to the main Fortran program. This subroutine is a modification of the "Matrix Equation Solver" Fortran extended subroutine due to the Computer Science Center, Wright-Patterson AFB, Ohio. The values of the linear coefficients and constants are tabled for each N and M. In addition, the value of the mean square error function is tabled for each sample of size N and M. The values of the mean square error functions are calculated from Equations (49) and (54).

Equation (49) can be rearranged to give:

$$MSE = \sum_{i=3}^{N-2} \sum_{j=3}^{N-2} a_i a_j \sigma_{ij} + [\sum_{i=3}^{N-2} a_i \mu_i - A]^2 \qquad (59)$$

and a similar expression can be developed for the MSE of the scale parameter. With the known values of a_i and A from the solution of Equation (57), the MSE is easily computed in a subroutine to the main program.

Additional Censoring

Censoring from Above. The minimally censored sample was defined as the basic ordered sample of size N with the two extreme order statistics censored from each end of the sample. Additional censoring was accomplished by censoring the sample from above so that M (the number of sample values remaining after censoring) decreases from (N-4) to (1) for each sample size (N). The value of the mean square error function was also tabled for each M. The values of the coefficients and MSE for the minimally censored sample and

additional censoring from above are given in Table I, Appendix A.

Symmetric Censoring. Each sample of size N=5(1)20 was also censored symmetrically from both ends. In this case M ranges from N-4 to 2 for even sample sizes and N-4 to 3 for odd samples. The symmetric censoring was terminated at M=3 since with M=1 the estimate of the location parameter is the median and there is no information available to compute the estimate of the scale parameter.

V. Use of the Tables

Introduction

The results of this thesis are Tables I and II included in the Appendices. With these tables the user is able to calculate the best linear invariant estimates of the location and scale parameters of the Cauchy distribution. This chapter explains these tables, gives the required procedure to obtain the conditional estimates or simultaneous estimates, and provides examples of these calculations.

Explanation of Tables I and II

Both tables include the values of the mean square error function, and the values of the coefficients and constants which are applied to the sample data to obtain the estimates of the parameters. These values are tabled for each N (sample size) and M (sample size after censoring). The coefficients of the order statistics required for estimation of the location parameter are listed in the columns under **LOCATION**, and the same values for the scale parameter are listed in the same manner under **SCALE**. Table I is used for a minimally censored sample or for a sample with additional censoring from above, and Table II is used for a sample which has been censored symmetrically.

Estimation Procedure

Best linear invariant estimation of a parameter consists of the following steps:

1. Obtain the sample data.
2. Order the data and determine N.
3. Minimally censor the two extreme sample values from each end, and additionally censor as desired.
4. Determine N and M and enter the appropriate table to obtain the coefficients and constants.
5. Multiply the sample values by their respective coefficients and sum the result to obtain the estimate of the parameter.
6. If the sample was additionally censored from above (Table I) conditional estimation is required, and the appropriate constant times the known parameter must be summed with the terms in step 5.

Examples

No Additional Censoring. As an example of the use of the tables to determine an estimate of the parameters of the Cauchy distribution, assume that the following data are known to come from a Cauchy distributed parent population. The true value of the location parameter is 8.0 and the true value of the scale parameter is .5.

$x_1 = 5.689853$	$x_6 = 9.222433$
$x_2 = 7.835235$	$x_7 = 8.316519$
$x_3 = 9.641365$	$x_8 = 7.902609$
$x_4 = 7.201119$	$x_9 = 18.926210$
$x_5 = 8.739464$	

When the data are ordered the following order statistics are obtained:

$$x_{(1)} = 5.689853 \qquad x_{(6)} = 8.739464$$
$$x_{(2)} = 7.201119 \qquad x_{(7)} = 9.222433$$
$$x_{(3)} = 7.835235 \qquad x_{(8)} = 9.641365$$
$$x_{(4)} = 7.902609 \qquad x_{(9)} = 18.926210$$
$$x_{(5)} = 8.316519$$

After these statistics are censored the subset $x_{(3)}$ through $x_{(7)}$ remains with N=9 and M=5. From Table I, the following coefficients are obtained:

$$a_3 = -.067277 \qquad a_6 = .245395$$
$$a_4 = .245395 \qquad a_7 = -.067277$$
$$a_5 = .643765 \qquad A = 0$$

Now the estimate of the location parameter t* is

$$t^* = a_3x_{(3)}+a_4x_{(4)}+a_5x_{(5)}+a_6x_{(6)}+a_7x_{(7)} \tag{60}$$

And when this calculation is carried out t* = 8.290177.

To estimate the scale parameter the required coefficients are obtained from the same table.

$$d_3 = -.153945 \qquad d_6 = .369546$$
$$d_4 = -.369546 \qquad d_7 = .153945$$
$$d_5 = 0 \qquad D = 0$$

Now

$$s^* = d_3x_{(3)}+d_4x_{(4)}+d_5x_{(5)}+d_6x_{(6)}+d_7x_{(7)} \tag{61}$$

and when these calculations are carried out s* = .522809.

It can be seen by inspecting the tables that the estimation is conditional estimation when the sample is censored additionally from above. In all other cases the values of A and D are equal to zero.

Additional Censoring from Above. To demonstrate the conditional estimation procedure, consider the data from the previous example. Additional censoring from above will be performed so that $N=9$ and $M=3$. Now from Table I:

$$a_3 = -.070500 \qquad a_5 = .82517$$

$$a_4 = .245330 \qquad A = -.032654$$

and

$$d_3 = -.231235 \qquad d_5 = .779717$$

$$d_4 = -.588770 \qquad D = -.040287$$

$$t^*|s = a_3x_{(3)}+a_4x_{(4)}+a_5x_{(5)}-As \tag{62}$$

$$s^*|t = d_3x_{(3)}+d_4x_{(4)}+d_5x_{(5)}-Dt \tag{63}$$

When these calculations are carried out, the following estimates are obtained: $t^*|s = 8.265232$ and $s^*|t = .342227$. The same procedures apply to the estimation of the parameters with additional symmetric censoring with the coefficients from Table II. The reader will notice that the value of the MSE of the estimate increases as less information is considered. For the location parameter estimator of the last example the MSE increases from .38655 to .39752 as M decreases from 5 to 3.

Simultaneous Estimation. Due to a technique suggested by Herman for unbiased estimation (Ref 8), it is possible to use Table I for simultaneous estimation of the parameters when neither is known.

If $\bar{t}$ = the simultaneous estimate of the location parameter

$\bar{s}$ = the simultaneous estimate of the scale parameter

t^* and s^* are as defined earlier

then
$$\bar{t} = \frac{\sum_{i=3}^{M+2} a_i x_{(i)} - A \sum_{i=3}^{M+2} d_i x_{(i)}}{1-AD} \tag{64}$$

and
$$\bar{s} = \frac{\sum_{i=3}^{M+2} d_i x_{(i)} - D \sum_{i=3}^{M+2} a_i x_{(i)}}{1-AD} \tag{65}$$

As an example of this technique, consider the data from the example with "additional censoring from above".

$a_3 = -.070500 \qquad d_3 = -.231235 \qquad x_{(3)} = 7.835235$

$a_4 = .245330 \qquad d_4 = -.588770 \qquad x_{(4)} = 7.902609$

$a_5 = .82517 \qquad d_5 = .779717 \qquad x_{(5)} = 8.316519$

$A = -.032654 \qquad D = -.040287$

$$\bar{t} = \frac{8.248905 + .000651}{.998684} = 8.260428$$

$$\bar{s} = \frac{.019931 + .332324}{.998684} = .352719$$

In summary, it can be seen that although this technique of estimation is conditional estimation, the conditional requirement is only necessary when estimating from sample data

that have been censored additionally from above. The method provides a simple and efficient procedure to estimate the Cauchy parameters.

VI. Summary

The objective of this thesis was to develop a table of linear coefficients which could be easily applied to sample data to obtain the conditional best linear invariant estimates of the location and scale parameters of the Cauchy distribution. These estimates are best in the sense that they process minimum mean square error. The coefficients and constants required to calculate these estimates for minimally censored samples and samples with additional censoring from above are tabled in Appendix A. The same values for samples with additional symmetric censoring from above and below are tabled in Appendix B. In addition the value of the mean square error function for each sample size is also included in these tables. The computer programs required to calculate and table the above data are included in Appendix C.

This paper also includes a brief review of the previous work in estimation of the parameters of the Cauchy distribution from 1961 through 1970. Much of this effort was accomplished at the Air Force Institute of Technology under the direction of Professor Albert H. Moore and sponsorship of Dr. H. Leon Harter. These works are concerned primarily with unbiased estimates of the parameters.

The Cauchy distribution and its peculiarities are discussed with a review of several ways in which the distribution may be generated. The order statistic theory required

to obtain the expected values, variance, and covariances of the order statistics is also reviewed. The mean square error function for the estimate of the parameters is developed and minimized by Lagrangian techniques to obtain the matrix equations required to calculate the linear coefficients.

The report is concluded with an explanation of the tables and several examples of the application of these coefficients. A technique of simultaneous estimation of both parameters of the distribution is also presented with an example of the technique.

The method of estimation presented in this paper and the attached tables of linear coefficients provide a simple and efficient method of obtaining either conditional or simultaneous best linear invariant estimates of the parameters of the Cauchy distribution.

Bibliography

1. Barnett, V. D. "Order Statistics Estimation of the Location of the Cauchy Distribution". J. American Statistical Association, 61: 1205-1218 (1966).

2. Chamberlain, R. B. Nearly Best Linear Unbiased Estimators of the Location and Scale Parameters of the Cauchy Distribution by the Use of Censored Order Statistics. Unpublished thesis. Wright-Patterson Air Force Base, Ohio, Air Force Institute of Technology, 1966.

3. Chan, L. K. "Linear Estimation of the Location and Scale Parameters of the Cauchy Distribution Based on Sample Quantities". J. American Statistical Association, 65: 851-859 (1970).

4. Cramer, H. Mathematical Methods of Statistics. Princeton: Princeton University Press, 1946.

5. Feller, W. An Introduction to Probability Theory and its Applications, Vol II. New York: John Wiley and Sons, 1966.

6. Freund, J. E. Mathematical Statistics. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962.

7. Gray, R. M. Nearly Best Linear Invariant Conditional Estimation of the Scale and Location Parameters of the Gama Probability Distribution. Unpublished thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1970.

8. Herman, W. J. Least Squares Conditional Estimation of Weibull Populations. Unpublished thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1966.

9. Jonson, E. C. Conditional Nearly Best Linear Estimation of the Location and Scale Parameters of the Cauchy Distribution by the Use of Censored Order Statistics. Unpublished thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1969.

10. Meyer, P. L. Introductory Probability and Statistical Applications. Reading, Massachusetts: Addison-Wesley Publishing Company, 1970.

11. Mood, A. M. and F. A. Graybill. Introduction to the Theory of Statistics. New York, N. Y.: McGraw-Hill Book Company, 1963.

12. Mosteller, F. "On Some Useful 'Inefficient' Statistics". Annals of Mathematical Statistics, 17: 377-408 (1946).

13. Rider, Paul R. "Variance of the Median of Samples from a Cauchy Distribution". J. American Statistical Association, 55: 322-323 (1960).

14. Rothenberg, T. J., F. M. Fisher, and C. B. Tilanus. "A Note on Estimation from a Cauchy Sample". J. American Statistical Association, 59: 460-463 (1964).

15. Sarhan, A. E. and B. G. Greenberg. Contributions to Order Statistics. New York, N. Y.: John Wiley and Sons, 1962.

16. Stark, T. M. Simultaneous and Conditional Estimation of the Location and Scale Parameters of the Cauchy Distribution by the Use of Selected Order Statistics. Unpublished thesis. Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, 1966.

17. Taylor, A. E. Advanced Calculus. Boston, Massachusetts: Ginn and Company, 1955.

APPENDIX A

Table I

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
5	1	1.22125	3	1.000000	1.00000	3	0.000000
			A	0.000000		D	0.000000
6	2	.86657	3	.500000	.63722	3	-.501421
			4	.500000		4	.501421
			A	0.000000		D	0.000000
6	1	1.59039	3	1.000000	.89285	3	-.296212
			A	-.351750		D	-.296212
7	3	.60725	3	.059194	.56682	3	-.394255
			4	[illegible]		4	.000000
			5	.059194		5	.394255
			A	[illegible]		D	.000000
7	2	.60955	3	.954057	.64998	3	-.552955
			4	.945343		4	.516027
			A	-.334301		D	-.035878
7	1	1.27736	3	1.000000	.76118	3	-.377243
			A	-.633763		D	-.377243
8	4	.47326	3	-.050159	.40649	3	-.245497
			4	.550159		4	-.342910
			5	.550159		5	.342910
			6	-.050159		6	.245497
			A	.000000		D	-.000000
8	3	.47544	3	-.049331	.46637	3	-.297036
			4	.557989		4	-.423857
			5	.491342		5	.736492
			A	.026304		D	.025803
8	2	.54623	3	-.078387	.66325	3	-.449491
			4	1.078387		4	.209532
			A	-.137294		D	-.239559

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
8	1	1.56887	3	1.000000	.67699	3	-.373372
			A	-.865386		D	-.373372
9	5	.38655	3	-.067277	.34136	3	-.153946
			4	.245395		4	-.369546
			5	.643765		5	-.000000
			6	.245395		6	.369546
			7	-.067277		7	.153946
			A	-.000000		D	.000000
9	4	.39134	3	-.067719	.36208	3	-.166986
			4	.261151		4	-.424693
			5	.657829		5	-.014642
			6	.158829		6	.616926
			A	.032798		D	.071903
9	3	.39752	3	-.072500	.49045	3	-.231235
			4	.245337		4	-.568779
			5	.825170		5	.779717
			A	-.032654		D	-.041297
9	2	.56204	3	-.125200	.62227	3	-.315027
			4	1.125200		4	-.087019
			A	-.753133		D	-.402044
9	1	1.92972	3	1.000000	.62466	3	-.348568
			A	-1.976808		D	-.348568
10	6	.72626	3	-.062304	.29371	3	-.100113
			4	.033953		4	-.303114
			5	.478052		5	-.212155
			6	.478052		6	.212155
			7	.033953		7	.303114
			8	-.062304		8	.100113
			A	-.000000		D	0.000000

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COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
10	5	.33118	3	-.062798	.30691	3	-.104844
			4	.036164		4	-.318302
			5	.498000		5	-.226247
			6	.499474		6	.214673
			7	-.009849		7	.459766
			A	.027727		C	.025046
10	4	.33118	3	-.062795	.37224	3	-.129212
			4	.086182		4	-.398429
			5	.499049		5	-.306690
			6	.498864		6	.865059
			A	.027748		C	.030738
10	3	.36670	3	-.073197	.51050	3	-.185192
			4	.075813		4	-.590117
			5	.027681		5	.531655
			A	-.139110		C	-.193624
10	2	.61474	3	-.146186	.56628	3	-.214501
			4	1.146186		4	-.261139
			A	-.516339		C	-.475640
10	1	2.34957	3	1.950300	.59086	3	-.320762
			A	-1.275012		C	-.320762
11	7	.28197	3	-.052380	.25743	3	-.067913
			4	.018550		4	-.231556
			5	.295622		5	-.275726
			6	.496199		6	-.000000
			7	.295622		7	.275726
			8	.008659		8	.231556
			9	-.052380		9	.067913
			A	-.000000		C	-.000000

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COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
11	6	.25616	3	-.353106	.26450	3	-.069538
			4	.009127		4	-.238771
			5	.311130		5	-.284794
			6	.535658		6	-.002774
			7	.312752		7	.279757
			8	-.065558		8	.335071
			A	.021011		D	.019421
11	5	.28737	3	-.053233	.30034	3	-.079526
			4	.059779		4	-.274159
			5	.314193		5	-.333652
			6	.510981		6	-.021313
			7	.228280		7	.755317
			A	.044547		D	.046557
11	4	.29404	3	-.055050	.39377	3	-.106737
			4	.038670		4	-.373581
			5	.302151		5	-.476005
			6	.746029		6	.919502
			A	-.027962		D	-.037231
11	3	.36440	3	-.072501	.50400	3	-.142559
			4	-.014380		4	-.565973
			5	1.036881		5	.296992
			A	-.254526		D	-.352037
11	2	.68974	3	-.157469	.51717	3	-.148769
			4	1.157469		4	-.344726
			A	-.658171		D	-.497405
11	1	2.82441	3	1.000000	.56795	3	-.294753
			A	-1.465911		D	-.294753

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
12	8	.24810	3	-.043156	.22895	3	-.047234
			4	-.024229		4	-.174151
			5	.164418		5	-.266692
			6	.432968		6	-.138924
			7	.432968		7	.138924
			8	.164418		8	.266692
			9	-.024229		9	.174151
			10	-.043156		10	.047234
			A	.000000		C	.000000
12	7	.25146	3	-.043718	.23299	3	-.048090
			4	-.024417		4	-.177375
			5	.157102		5	-.271901
			6	.439403		6	-.142455
			7	.439907		7	.139738
			8	.168759		8	.259507
			9	-.083635		9	.245441
			A	.016644		D	.014866
12	6	.25385	3	-.044 74	.25368	3	-.052546
			4	-.024281		4	-.194254
			5	.169844		5	-.299573
			6	.415629		6	-.162238
			7	.417115		7	.142029
			8	.065767		8	.611014
			A	.045201		C	.045231
12	5	.25440	3	-.044239	.31139	3	-.065303
			4	-.024761		4	-.247095
			5	.169002		5	-.381476
			6	.414241		6	-.225124
			7	.485777		7	.946081
			A	.025297		C	.031082
12	4	.27609	3	-.049964	.41165	3	-.058587
			4	-.032 63		4	-.333771
			5	.169986		5	-.578535
			6	.712.41		6	.799603
			A	-.108151		C	-.161253

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
12	3	.37871	3	-.371378	.47849	3	-.106153
			4	-.069712		4	-.434062
			5	1.137290		5	.546653
			A	-.366892		C	-.463562
12	2	.78122	3	-.164384	.47882	3	-.106516
			4	1.164384		4	-.378495
			A	-.791721		C	-.485011
12	1	7.75236	3	1.000000	.55178	3	-.271615
			A	-1.650222		C	-.271615
13	9	.22140	3	-.035397	.20604	3	-.033978
			4	-.037060		4	-.131415
			5	.031475		5	-.232913
			6	.290384		6	-.201347
			7	.401195		7	.000000
			8	.290384		8	.201347
			9	.031475		9	.232913
			10	-.037060		10	.131415
			11	-.035397		11	.033978
			A	0.000000		C	.000000
13	8	.22403	3	-.035908	.20846	3	-.034358
			4	-.037441		4	-.133021
			5	.082045		5	-.235753
			6	.294295		6	-.204156
			7	.416728		7	-.009778
			8	.294858		8	.202741
			9	.083571		9	.234467
			10	-.038859		10	.132257
			A	.012274		C	.011422

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
13	7	.22698	3	-.036231	.22099	3	-.036523
			4	-.037726		4	-.141427
			5	.024307		5	-.251238
			6	.299768		6	-.219293
			7	.414056		7	-.005401
			8	.301725		8	.205084
			9	-.025098		9	.464178
			A	.040426		C	.039378
13	6	.22696	3	-.076231	.25686	3	-.042739
			4	-.037605		4	-.155044
			5	.094066		5	-.297105
			6	.239862		6	-.255517
			7	.414977		7	-.322553
			8	.274446		8	.847945
			A	.047681		C	.053962
13	5	.27351	3	-.037424	.32890	3	-.055599
			4	-.139023		4	-.217447
			5	.397147		5	-.394615
			6	.703764		6	-.367904
			7	.653497		7	1.001928
			A	-.023931		C	-.037678
13	4	.27132	3	-.044687	.41570	3	-.072001
			4	-.352220		4	-.287591
			5	.080587		5	-.524221
			6	1.016415		6	.579841
			A	-.195043		C	-.300382
13	3	.41790	3	-.373399	.44695	3	-.378999
			4	-.397513		4	-.713353
			5	1.167912		5	-.131819
			A	-.473784		C	-.524168
13	2	.88646	3	-.169347	.44967	3	-.378745
			4	1.169047		4	-.386951
			A	-.917946		C	-.465646

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	** LOCATION ** MSE	I	COEF.	** SCALE ** MSE	I	COEF.
13	1	3.93247	3	1.000000	.57998	3	-.251330
			A	-1.830725		C	-.251330
14	10	.19961	3	-.029130	.18722	3	-.029067
			4	-.040524		4	-.100183
			5	.031712		5	-.194902
			6	.195010		6	-.215977
			7	.342931		7	-.096717
			8	.342931		8	.096717
			9	.195010		9	.215977
			10	.031712		10	.194902
			11	-.040524		11	.100183
			12	-.029130		12	.029067
			A	-.000000		C	-.000000
14	9	.20187	3	-.029426	.18874	3	-.025575
			4	-.040914		4	-.101002
			5	.032132		5	-.196569
			6	.197279		6	-.217620
			7	.346861		7	-.097840
			8	.347041		8	.097013
			9	.197726		9	.217029
			10	.032799		10	.195876
			11	-.093458		11	.137516
			A	.009475		C	.008858
14	8	.21470	3	-.029822	.19665	3	-.026392
			4	-.041391		4	-.106422
			5	.032898		5	-.205324
			6	.200724		6	-.228155
			7	.352978		7	-.104043
			8	.353668		8	.098107
			9	.202570		9	.222605
			10	-.071674		10	.381067
			A	.034352		C	.033700

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
14	7	.20543	3	-.029909	.21956	3	-.029550
			4	-.041427		4	-.118360
			5	.033365		5	-.231309
			6	.202193		6	-.299260
			7	.355501		7	-.123714
			8	.356634		8	.099797
			9	.123649		9	.720625
			A	.054135		C	.057660
14	6	.20661	3	-.030130	.25827	3	-.076455
			4	-.041029		4	-.146551
			5	.032729		5	-.298443
			6	.231616		6	-.329041
			7	.754758		7	-.170684
			8	.492966		8	1.031792
			A	.023116		C	.030014
14	5	.22120	3	-.032504	.34375	3	-.047514
			4	-.046649		4	-.192090
			5	.030123		5	-.382069
			6	.205303		6	-.446757
			7	.947412		7	.970685
			A	-.388665		C	-.177774
14	4	.27538	3	-.041681	.40631	3	-.057335
			4	-.063835		4	-.237098
			5	.021299		5	-.469446
			6	1.094217		6	.339976
			A	-.293381		C	-.419073
14	3	.43701	3	-.069627	.41657	3	-.059407
			4	-.118676		4	-.242495
			5	1.138273		5	-.246471
			A	-.576769		C	-.548457
14	2	1.06416	3	-.172420	.49747	3	-.059996
			4	1.172420		4	-.382701
			A	-1.070659		C	-.442597

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
14	1	4.56418	3	1.030306	.57116	3	-.277584
			A	-2.037179		C	-.273844
15	11	.18201	3	-.024130	.17150	3	-.015911
			4	-.039711		4	-.077336
			5	.032746		5	-.150423
			6	.123831		6	-.206820
			7	.269319		7	-.180795
			8	.335888		8	[illegible]
			9	.269319		9	.180795
			10	.123831		10	.206829
			11	.032746		11	.150423
			12	-.039711		12	.077336
			13	-.024130		13	.015911
			A	.000009		C	[illegible]
15	10	.18363	3	-.024342	.17250	3	-.019[illegible]
			4	-.040150		4	-.077794
			5	.032816		5	-.151347
			6	.125041		6	-.207[illegible]
			7	.271917		7	-.181121
			8	.339193		8	-.[illegible]747
			9	.272174		9	.157630
			10	.125427		10	.207[illegible]
			11	.037298		11	.151937
			12	-.075433		12	.105402
			A	.007396		C	.016948
15	9	.16627	3	-.024678	.17766	3	-.019594
			4	-.040568		4	-.081196
			5	.033021		5	-.146407
			6	.127195		6	-.214255
			7	.276473		7	-.153677
			8	.345055		8	-.111740
			9	.277356		9	.157214
			10	.128764		10	.211002
			11	-.092619		11	.311002
			A	.029603		C	.127[illegible]

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
15	8	.18755	3	-.024839	.19275	3	-.021369
			4	-.040774		4	-.087292
			5	.002293		5	-.181476
			6	.128662		6	-.234450
			7	.279438		7	-.173487
			8	.348973		8	-.097074
			9	.231160		9	.159772
			10	.024987		10	.599783
			A	.053600		C	.054471
15	7	.18790	3	-.024851	.22562	3	-.025902
			4	-.040812		4	-.102995
			5	.003258		5	-.214926
			6	.128554		6	-.279924
			7	.279186		7	-.212549
			8	.348615		8	-.021025
			9	.316171		9	.913932
			A	.047162		C	.056721
15	6	.19283	3	-.025563	.28238	3	-.031787
			4	-.042485		4	-.130850
			5	.011718		5	-.274741
			6	.128729		6	-.302567
			7	.281414		7	-.286215
			8	.656767		8	1.055739
			A	-.029750		D	-.030386
15	5	.21673	3	-.020199	.35072	3	-.040111
			4	-.049671		4	-.165933
			5	-.013599		5	-.351061
			6	.133779		6	-.470708
			7	.049182		7	.769461
			A	-.150455		C	-.258041
15	4	.28561	3	-.039542	.38881	3	-.045200
			4	-.075989		4	-.187657
			5	-.019918		5	-.395469
			6	1.175269		6	.129179
			8	-.369382		D	-.502945

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
15	3	.47643	3	-.069066	.39032	3	-.045589
			4	-.133482		4	-.189475
			5	1.202547		5	-.315933
			A	-.572392		C	-.550856
15	2	1.17326	3	-.174992	.41034	3	-.046684
			4	1.174907		4	-.372402
			A	-1.157403		C	-.419086
15	1	5.24717	3	1.5[illegible]	.52440	3	-.218017
			A	-2.181482		C	-.218017
16	12	.16709	3	-.020142	.15819	3	-.014524
			4	-.037067		4	-.060477
			5	-.013593		5	-.131367
			6	.073662		6	-.186477
			7	.219627		7	-.172219
			8	.296514		8	-.070798
			9	.296514		9	.070798
			10	.219627		10	.172219
			11	.073662		11	.186477
			12	-.013593		12	.131367
			13	-.037067		13	.060477
			14	-.020142		14	.014524
			A	.000000		C	-.000000
16	11	.16837	3	-.020396	.15885	3	-.014586
			4	-.037344		4	-.060732
			5	-.013674		5	-.131935
			6	.074263		6	-.187319
			7	.202274		7	-.173019
			8	.238964		8	-.071222
			9	.290[illegible]		9	.070920
			10	.202474		10	.172721
			11	.074581		11	.187065
			12	-.017224		12	.171590
			13	-.066971		13	.092011
			A	.035842		C	.035612

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
16	10	.17067	3	-.020568	.16233	3	-.014911
			4	-.037827		4	-.062096
			5	-.013772		5	-.134932
			6	.075514		6	-.191654
			7	.205427		7	-.177287
			8	.303661		8	-.073539
			9	.303969		9	.071421
			10	.206279		10	.175217
			11	.076763		11	.189661
			12	-.099438		12	.240615
			A	.023650		D	.022495
16	9	.17273	3	-.020759	.17256	3	-.015874
			4	-.038143		4	-.066138
			5	-.017747		5	-.143839
			6	.076603		6	-.204560
			7	.208095		7	-.190167
			8	.307859		8	-.080834
			9	.308314		9	.072382
			10	.209948		10	.182021
			11	-.037969		11	.495708
			A	.048613		D	.048678
16	8	.17245	3	-.020768	.19509	3	-.018016
			4	-.038144		4	-.075148
			5	-.013679		5	-.163766
			6	.076829		6	-.233952
			7	.208549		7	-.219615
			8	.308338		8	-.098325
			9	.309125		9	.072809
			10	.169729		10	.301061
			A	.057497		D	.065547

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
16	7	.17397	3	-.020982	.23591	3	-.021953
			4	-.038640		4	-.091766
			5	-.014272		5	-.200705
			6	.076477		6	-.288777
			7	.208592		7	-.275791
			8	.308432		8	-.133829
			9	.430397		9	1.041716
			A	.021016		D	.028499
16	6	.18459	3	-.022393	.20460	3	-.027751
			4	-.041581		4	-.116340
			5	-.017155		5	-.255769
			6	.076974		6	-.371491
			7	.214086		7	-.362846
			8	.793269		8	1.014156
			A	-.375217		D	-.127043
16	5	.21736	3	-.026756	.34888	3	-.033756
			4	-.051020		4	-.140286
			5	-.025734		5	-.310036
			6	.079083		6	-.454754
			7	1.024427		7	.567076
			A	-.231331		D	-.371305
16	4	.30046	3	-.037976	.36833	3	-.035637
			4	-.075517		4	-.150212
			5	-.049359		5	-.333147
			6	1.162852		6	-.074632
			A	-.451610		D	-.553629
16	3	.52125	3	-.068644	.36844	3	-.035801
			4	-.144795		4	-.150028
			5	1.213028		5	-.355550
			A	-.765624		D	-.541179
16	2	1.27356	3	-.177034	.39694	3	-.037151
			4	1.177034		4	-.350354
			A	-1.272158		D	-.396514

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
16	1	5.98125	3	1.000065	.51914	3	-.204294
			A	-2.353784		D	-.204294
17	13	.15440	3	-.016948	.14677	3	-.011348
			4	-.033775		4	-.047878
			5	-.022361		5	-.107651
			6	.039373		6	-.163919
			7	.143717		7	-.175151
			8	.245754		8	-.115015
			9	.288461		9	.000000
			10	.245754		10	.115015
			11	.143717		11	.175151
			12	.039373		12	.163919
			13	-.022361		13	.107651
			14	-.033775		14	.047878
			15	-.016948		15	.011348
			A	.000000		D	.000000
17	12	.15543	3	-.017560	.14723	3	-.011384
			4	-.033995		4	-.048030
			5	-.022406		5	-.107995
			6	.039664		6	-.164450
			7	.144729		7	-.175736
			8	.247461		8	-.115439
			9	.290527		9	-.000096
			10	.247572		10	.115247
			11	.144398		11	.175543
			12	.039992		12	.164257
			13	-.022238		13	.107806
			14	-.058587		14	.064698
			A	.004668		C	.004422

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
17	11	.15742	3	-.017275	.14963	3	-.011572
			4	-.034414		4	-.048830
			5	-.022735		5	-.109811
			6	.040287		6	-.167258
			7	.146962		7	-.178946
			8	.251008		8	-.117718
			9	.294774		9	-.030673
			10	.251412		10	.116387
			11	.147567		11	.177536
			12	.041281		12	.165969
			13	-.098682		13	.193391
			A	.019544		D	.018577
17	10	.15920	3	-.017465	.15677	3	-.012134
			4	-.034777		4	-.051213
			5	-.022896		5	-.115222
			6	.040969		6	-.175658
			7	.148893		7	-.188201
			8	.254536		8	-.124689
			9	.299082		9	-.002656
			10	.255471		10	.119465
			11	.150622		11	.183114
			12	-.074439		12	.409684
			A	.043159		C	.042490
17	9	.15968	3	-.017512	.17250	3	-.013338
			4	-.034947		4	-.056545
			5	-.022872		5	-.127365
			6	.041700		6	-.194587
			7	.149717		7	-.209431
			8	.255902		8	-.140899
			9	.300799		9	-.007914
			10	.257255		10	.125352
			11	.173257		11	.689583
			A	.059947		C	.064757

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
17	8	.15994	3	-.017547	.29170	3	-.015736
			4	-.034941		4	-.066547
			5	-.023022		5	-.150219
			6	.041117		6	-.230395
			7	.149504		7	-.250117
			8	.255620		8	-.172748
			9	.310354		9	-.019956
			10	.228916		10	.962805
			A	.045277		D	.057087
17	7	.16441	3	-.018186	.24740	3	-.019479
			4	-.036169		4	-.082904
			5	-.024409		5	-.185852
			6	.042640		6	-.288277
			7	.150923		7	-.316774
			8	.258506		8	-.226763
			9	.628695		9	1.093044
			A	-.018338		D	-.027595
17	6	.18066	3	-.020022	.30198	3	-.024051
			4	-.040501		4	-.102164
			5	-.029017		5	-.272305
			6	.048009		6	-.367853
			7	.157725		7	-.402131
			8	.891902		8	.990870
			A	-.174389		D	-.224874
17	5	.22184	3	-.024969	.34019	3	-.027439
			4	-.051610		4	-.116824
			5	-.040919		5	-.266581
			6	.038907		6	-.416689
			7	1.079393		7	.364189
			A	-.302156		D	-.463353
17	4	.31695	3	-.036805	.34896	3	-.028794
			4	-.078716		4	-.121659
			5	-.071319		5	-.275744
			6	1.136942		6	-.154522
			A	-.570738		D	-.572169

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
17	3	.57089	3	-.068335	.35046	3	-.028309
			4	-.152717		4	-.120576
			5	1.221052		5	-.376428
			A	-.855725		B	-.525314
17	2	1.42468	3	-.178705	.38639	3	-.030092
			4	1.178705		4	-.345311
			A	-1.384401		B	-.375393
17	1	6.76628	3	1.000000	.51497	3	-.192132
			A	-2.524459		B	-.192132
18	14	.14349	3	-.014372	.13687	3	-.008997
			4	-.030399		4	-.038353
			5	-.026609		5	-.088544
			6	.016373		6	-.142010
			7	.099335		7	-.167494
			8	.195338		8	-.138345
			9	.260334		9	-.053884
			10	.260334		10	.053884
			11	.195338		11	.138345
			12	.099335		12	.167494
			13	.016373		13	.142010
			14	-.026609		14	.088544
			15	-.030399		15	.038353
			16	-.014372		16	.008997
			A	.000000		B	-.000000

TABLE T

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
18	13	.14432	3	-.014454	.13719	3	-.009319
			4	-.030571		4	-.038445
			5	-.026755		5	-.088757
			6	.016484		6	-.142365
			7	.099040		7	-.167969
			8	.196517		8	-.139706
			9	.261915		9	-.054954
			10	.261946		10	.053938
			11	.196605		11	.178580
			12	.100678		12	.167781
			13	.016559		13	.143224
			14	-.026562		14	.068628
			15	-.051802		15	.051689
			A	.003771		C	.003585
18	12	.14602	3	-.014623	.13839	3	-.009172
			4	-.030923		4	-.038929
			5	-.027042		5	-.089883
			6	.016744		6	-.144180
			7	.101247		7	-.170113
			8	.199148		8	-.140633
			9	.265324		9	-.055652
			10	.265458		10	.054184
			11	.199453		11	.139765
			12	.101966		12	.169249
			13	.017624		13	.143315
			14	-.094077		14	.156816
			A	.016197		C	.015406

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
18	11	.14779	3	-.014797	.14393	3	-.009470
			4	-.031279		4	-.049376
			5	-.027310		5	-.093249
			6	.017386		6	-.149553
			7	.102740		7	-.176740
			8	.201904		8	-.146490
			9	.269173		9	-.058120
			10	.269524		10	.054714
			11	.202885		11	.143118
			12	.104254		12	.173431
			13	-.094180		13	.339527
			A	.037675		C	.036691
18	10	.14860	3	-.014873	.15519	3	-.010228
			4	-.031426		4	-.047826
			5	-.027787		5	-.110823
			6	.017346		6	-.162051
			7	.103612		7	-.191764
			8	.203510		8	-.159890
			9	.271372		9	-.065462
			10	.271929		10	.055393
			11	.205162		11	.159025
			12	.030815		12	.589086
			A	.058111		C	.067689
18	9	.14860	3	-.014873	.17627	3	-.011659
			4	-.031426		4	-.049770
			5	-.027388		5	-.115169
			6	.017344		6	-.185479
			7	.103508		7	-.227577
			8	.203505		8	-.185924
			9	.271365		9	-.083446
			10	.271919		10	.055266
			11	.205945		11	.862454
			A	.057945		C	.068735

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
18	8	.15026	3	-.015357	.21065	3	-.014020
			4	-.031868		4	-.059926
			5	-.027965		5	-.138972
			6	.016945		6	-.224587
			7	.103661		7	-.268915
			8	.224084		8	-.270559
			9	.272101		9	-.197483
			10	.479078		10	1.071359
			A	.019186		C	.026898
18	7	.15830	3	-.015923	.25751	3	-.017296
			4	-.033883		4	-.074063
			5	-.039758		5	-.172220
			6	.015387		6	-.274468
			7	.109751		7	-.377784
			8	.209735		8	-.295567
			9	.748794		9	1.073302
			A	-.065339		C	-.106200
18	6	.18004	3	-.018267	.30355	3	-.020016
			4	-.039307		4	-.088445
			5	-.036748		5	-.206297
			6	.013355		6	-.336649
			7	.112076		7	-.410314
			8	.968889		8	.733574
			A	-.194981		C	-.328739
18	5	.22928	3	-.023614	.32746	3	-.022468
			4	-.051825		4	-.096845
			5	-.051758		5	-.225727
			6	.036668		6	-.369757
			7	1.129528		7	.184499
			A	-.371112		C	-.530829
18	4	.24050	3	-.035909	.32950	3	-.022702
			4	-.081149		4	-.097663
			5	-.088122		5	-.228702
			6	1.205080		6	-.238758
			A	-.667064		D	-.587454

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
18	3	.62501	3	-.058111	.73570	3	-.022887
			4	-.153259		4	-.098286
			5	1.227373		5	-.385447
			A	-.943231		D	-.506621
18	2	1.58643	3	-.130165	.37773	3	-.024748
			4	1.190166		4	-.331158
			A	-1.494767		D	-.355906
18	1	7.60217	3	1.000000	.51163	3	-.181294
			A	-2.697790		D	-.181294
19	15	.13431	3	-.013278	.12821	3	-.007227
			4	-.027194		4	-.031059
			5	-.028169		5	-.073208
			6	.031906		6	-.122162
			7	.035843		7	-.154747
			8	.155509		8	-.147151
			9	.223892		9	-.090459
			10	.252657		10	.000000
			11	.223902		11	.090459
			12	.155506		12	.147151
			13	.065843		13	.154747
			14	.031206		14	.122162
			15	-.028166		15	.073208
			16	-.027194		16	.031059
			17	-.013278		17	.007227
			A	-.000000		D	.397100

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	14	.17468	3	-.012379	.12844	3	-.007241
			4	-.027329		4	-.031116
			5	-.028324		5	-.073343
			6	.001922		6	-.122389
			7	.066192		7	-.155038
			8	.151377		8	-.147477
			9	.224863		9	-.090653
			10	.253993		10	-.000943
			11	.224903		11	.099568
			12	.151457		12	.147359
			13	.066307		13	.154959
			14	.001357		14	.122298
			15	-.028179		15	.073252
			16	-.045495		16	.041774
			A	.007377		C	.032934
19	13	.17613	3	-.012471	.12967	3	-.007311
			4	-.027618		4	-.031417
			5	-.028612		5	-.074056
			6	.001273		6	-.123538
			7	.066982		7	-.156584
			8	.153177		8	-.148958
			9	.227479		9	-.091692
			10	.256999		10	-.000291
			11	.227572		11	.091112
			12	.153560		12	.148375
			13	.067488		13	.155992
			14	.001885		14	.122993
			15	-.037716		15	.128271
			A	.013483		C	.012843

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	12	.17781	3	-.012622	.13332	3	-.007523
			4	-.027947		4	-.032323
			5	-.028929		5	-.076196
			6	.031376		6	-.127198
			7	.067977		7	-.161236
			8	.155306		8	-.153565
			9	.230700		9	-.094886
			10	.260732		10	-.001129
			11	.231191		11	.092629
			12	.156229		12	.151316
			13	.069256		13	.159001
			14	-.172263		14	.292652
			A	.032613		C	.031556
19	11	.17885	3	-.012714	.14151	3	-.037991
			4	-.028149		4	-.034354
			5	-.029194		5	-.081028
			6	.031505		6	-.135352
			7	.058710		7	- 171789
			8	.156847		8	-.164072
			9	.232984		9	-.102293
			10	.263475		10	-.003332
			11	.233879		11	.095730
			12	.158459		12	.157572
			13	-.045920		13	.532131
			A	.054159		D	.055195
19	10	.13806	3	-.012723	.15694	3	-.038885
			4	-.028154		4	-.039217
			5	-.029091		5	-.099219
			6	.031576		6	-.150957
			7	.068868		7	-.191991
			8	.157173		8	-.184334
			9	.233466		9	-.116939
			10	.264027		10	-.008265
			11	.234489		11	.130624
			12	.110355		12	.760415
			A	.053188		C	.071363

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	9	.13943	3	-.012771	.18263	3	-.010306
			4	-.028279		4	-.044712
			5	-.029276		5	-.105680
			6	.001379		6	-.177169
			7	.058756		7	-.226319
			8	.157114		8	-.219197
			9	.233423		9	-.142683
			10	.263843		10	-.018104
			11	.345912		11	1.000462
			A	.042918		C	.056216
19	8	.14725	3	-.013147	.22013	3	-.012605
			4	-.029182		4	-.054331
			5	-.030458		5	-.120670
			6	.000586		6	-.216769
			7	.069114		7	-.277968
			8	.159040		8	-.272248
			9	.236498		9	-.183250
			10	.607529		10	1.120277
			A	-.016450		C	-.025214
19	7	.15500	3	-.014292	.26448	3	-.015262
			4	-.031913		4	-.065967
			5	-.033931		5	-.156587
			6	-.001486		6	-.2640[illegible]
			7	.071019		7	-.341067
			8	.166276		8	-.378998
			9	.944325		9	.984616
			A	-.116143		C	-.198183
19	6	.18197	3	-.016932	.29991	3	-.017551
			4	-.038227		4	-.075061
			5	-.041991		5	-.179993
			6	-.006338		6	-.304879
			7	.075752		7	-.396482
			8	1.028136		8	.553752
			A	-.255298		C	-.423763

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	5	.27908	3	-.022581	.31367	3	-.018412
			4	-.051964		4	-.079684
			5	-.059701		5	-.189857
			6	-.317907		6	-.322359
			7	1.152113		7	.036786
			A	-.437972		C	-.573527
19	4	.76471	3	-.035212	.31316	3	-.018433
			4	-.082915		4	-.079783
			5	-.131308		5	-.190121
			6	1.219334		6	-.296745
			A	-.630935		C	-.584682
19	3	.68337	3	-.057951	.32254	3	-.018780
			4	-.164529		4	-.091173
			5	1.232480		5	-.387021
			A	-1.028575		C	-.486974
19	2	1.75568	3	-.181301	.37075	3	-.020622
			4	1.181301		4	-.317397
			A	-1.603411		D	-.339019
19	1	8.48386	3	1.000000	.51892	3	-.171584
			A	-2.862028		C	-.171584

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
20	16	.12560	3	-.010562	.12057	3	-.005874
			4	-.024268		4	-.025407
			5	-.028202		5	-.060984
			6	-.008602		6	-.104810
			7	.041053		7	-.140091
			8	.113176		8	-.146714
			9	.135798		9	-.112563
			10	.231607		10	-.042347
			11	.231604		11	.042339
			12	.135800		12	.112574
			13	.113181		13	.145715
			14	.041069		14	.140089
			15	-.008602		15	.104810
			16	-.028202		16	.060984
			17	-.024268		17	.025407
			18	-.010562		18	.005874
			A.	.000000		C	.000000
20	15	.12624	3	-.016568	.12174	3	-.005682
			4	-.024373		4	-.025443
			5	-.028323		5	-.060971
			6	-.008634		6	-.104961
			7	.041244		7	-.140205
			8	.113691		8	-.146931
			9	.186642		9	-.112744
			10	.232665		10	-.042428
			11	.232677		11	.042765
			12	.186668		12	.112691
			13	.113765		13	.145872
			14	.041329		14	.140230
			15	-.008529		15	.104896
			16	-.028212		16	.060907
			17	-.040321		17	.034119
			A	.012525		C	.002424

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
20	14	.12748	3	-.010711	.12164	3	-.005926
			4	-.024608		4	-.025635
			5	-.023590		5	-.061433
			6	-.008695		6	-.105760
			7	.041694		7	-.141376
			8	.114883		8	-.143089
			9	.188592		9	-.113684
			10	.235116		10	-.042893
			11	.235177		11	.042499
			12	.188792		12	.113290
			13	.115204		13	.147685
			14	.042182		14	.143959
			15	-.008217		15	.105734
			16	-.097744		16	.105954
			A	.011285		C	.019768
20	13	.12903	3	-.010844	.12433	3	-.006659
			4	-.024900		4	-.026212
			5	-.028914		5	-.062822
			6	-.008747		6	-.108169
			7	.042207		7	-.144640
			8	.116455		8	-.151593
			9	.191154		9	-.116552
			10	.238347		10	-.044329
			11	.239522		11	.042789
			12	.191676		12	.115019
			13	.117286		13	.150050
			14	.043368		14	.143377
			15	-.105714		15	.236547
			A	.028121		C	.027107

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM ABOVE)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
20	12	.13019	3	-.010976	.13039	3	-.006360
			4	-.025113		4	-.027517
			5	-.029140		5	-.065965
			6	-.038743		6	-.113631
			7	.042841		7	-.152042
			8	.117770		8	-.159595
			9	.193271		9	-.123144
			10	.241041		10	-.047752
			11	.241361		11	.043377
			12	.194240		12	.119683
			13	.119287		13	.155168
			14	-.075879		14	.428207
			A	.049336		C	.049410
20	11	.13052	3	-.010962	.14186	3	-.006971
			4	-.025166		4	-.030571
			5	-.029183		5	-.071959
			6	-.038697		6	-.124050
			7	.043085		7	-.166249
			8	.118287		8	-.174961
			9	.194091		9	-.135995
			10	.242094		10	-.054644
			11	.242539		11	.043634
			12	.195384		12	.125147
			13	.038526		13	.665765
			A	.063824		C	.069266
20	10	.13058	3	-.010968	.16115	3	-.007900
			4	-.025184		4	-.034212
			5	-.029215		5	-.082133
			6	-.038766		6	-.141792
			7	.043022		7	-.130470
			8	.118203		8	-.201413
			9	.193979		9	-.158349
			10	.241936		10	-.057265
			11	.242322		11	.043245
			12	.234649		12	.910571
			A	.056916		C	.070242

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	** LOCATION ** MSE	I	COEF.	** SCALE ** MSE	I	COEF.
20	9	.13224	3	-.011118	.19335	3	-.009379
			4	-.025558		4	-.040654
			5	-.029745		5	-.097727
			6	-.029205		6	-.169061
			7	.042924		7	-.227901
			8	.118672		8	-.242595
			9	.194953		9	-.193806
			10	.243159		10	-.088378
			11	.476018		11	1.094779
			A	.017609		D	.025346
20	8	.13855	3	-.011679	.22860	3	-.011345
			4	-.026932		4	-.049233
			5	-.031607		5	-.118533
			6	-.010622		6	-.205611
			7	.043209		7	-.278345
			8	.121509		8	-.298787
			9	.230297		9	-.247191
			10	.715825		10	1.109707
			A	-.057776		D	-.095326
20	7	.15392	3	-.013045	.26750	3	-.013399
			4	-.030266		4	-.058175
			5	-.036110		5	-.140335
			6	-.013945		6	-.244106
			7	.044132		7	-.332036
			8	.129322		8	-.359379
			9	.929211		9	.854776
			A	-.168407		D	-.292694
20	6	.18592	3	-.015965	.29250	3	-.014765
			4	-.037288		4	-.064222
			5	-.045674		5	-.155175
			6	-.021222		6	-.271593
			7	.045620		7	-.369521
			8	1.074467		8	.379477
			A	-.314576		D	-.494841

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	** LOCATION ** MSE	I	COEF.	** SCALE ** MSE	I	COEF.
20	5	.25085	3	-.021772	.29862	3	-.015159
			4	-.351823		4	-.065972
			5	-.085855		5	-.159556
			6	-.037460		6	-.278620
			7	1.175911		7	-.079225
			A	-.502787		C	-.598533
20	4	.39133	3	-.034662	.29903	3	-.015149
			4	-.094195		4	-.065919
			5	-.111898		5	-.159379
			6	1.230754		6	-.374709
			A	-.752676		C	-.575154
20	3	.74581	3	-.067941	.31345	3	-.015613
			4	-.159862		4	-.067849
			5	1.236762		5	-.383947
			A	-1.112198		C	-.467397
20	2	1.94133	3	-.182339	.36530	3	-.017382
			4	1.182339		4	-.034250
			A	-1.710676		C	-.321632
20	1	9.42671	3	1.330660	.50671	3	-.162840
			A	-3.329722		C	-.162840

APPENDIX B

Table II

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
5	1	1.22125	3	1.000000	1.00000	3	0.000000
			A	0.000000		D	0.000000
6	2	.86053	3	.500000	.63722	3	-.501421
			4	.500000		4	.501421
			A	0.000000		D	0.000000
7	3	.6[illegible]725	3	.059194	.50082	3	-.394255
			4	.881611		4	.000000
			5	.059194		5	.394255
			A	-.000000		D	.000000
8	4	.47326	3	-.050059	.40649	3	-.245497
			4	.550059		4	-.342910
			5	.550059		5	.342910
			6	-.050059		6	.245497
			A	.000000		D	-.000000
8	2	.47756	4	.500000	.55065	4	-.913836
			5	.500000		5	.913836
			A	0.000000		C	.000000
9	5	.38655	3	-.067277	.34138	3	-.153946
			4	.245395		4	-.369546
			5	.643765		5	-.000000
			6	.245395		6	.369546
			7	-.067277		7	.153946
			A	-.000000		C	.000000
9	3	.39619	4	.163962	.39972	4	-.678215
			5	.672076		5	-.000000
			6	.163962		6	.678215
			A	-.000000		D	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
10	6	.32626	3	-.062004	.29371	3	-.100113
			4	.083953		4	-.303114
			5	.478052		5	-.212155
			6	.478052		6	.212155
			7	.083953		7	.303114
			8	-.062004		8	.100113
			A	-.000000		C	0.000000
10	4	.33622	4	.000300	.32141	4	-.483478
			5	.499700		5	-.229589
			6	.499700		6	.229589
			7	.000300		7	.483478
			A	-.000000		C	.000000
10	2	.33622	5	.500000	.51926	5	-1.288071
			6	.500000		6	1.288071
			A	0.000000		C	.000000
11	7	.28197	3	-.052380	.25743	3	-.067613
			4	.008659		4	-.231556
			5	.295622		5	-.275726
			6	.496199		6	-.000000
			7	.295622		7	.275726
			8	.008659		8	.231556
			9	-.052380		9	.067613
			A	-.000000		C	-.000000
11	5	.29047	4	-.066116	.27198	4	-.345120
			5	.308387		5	-.289199
			6	.515459		6	.000000
			7	.308387		7	.289199
			8	-.066116		8	.345120
			A	.000000		C	.000000
11	3	.29292	5	.236954	.36217	5	-.932166
			6	.526093		6	.000000
			7	.236954		7	.932166
			A	-.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
12	8	.24810	3	-.043156	.22895	3	-.047234
			4	-.024229		4	-.174151
			5	.164418		5	-.266692
			6	.402968		6	-.138924
			7	.402968		7	.138924
			8	.164418		8	.266692
			9	-.024229		9	.174151
			10	-.043156		10	.047234
			A	.000000		C	.000000
12	6	.25491	4	-.037650	.23717	4	-.250641
			5	.171131		5	-.274964
			6	.416519		6	-.143749
			7	.416519		7	.143749
			8	.171131		8	.274964
			9	-.037650		9	.250641
			A	.000000		C	.000000
12	4	.25982	5	.069716	.28479	5	-.693568
			6	.430284		6	-.168750
			7	.430284		7	.168750
			8	.069716		8	.693568
			A	.000000		C	.000000
12	2	.26101	6	.500000	.50528	6	-1.643490
			7	.500000		7	1.643490
			A	0.000000		C	.000000
13	9	22140	3	-.035397	.20604	3	-.033978
			4	-.037060		4	-.171415
			5	.081475		5	-.232818
			6	.293384		6	-.201347
			7	.401196		7	.000000
			8	.293384		8	.201347
			9	.081475		9	.232818
			10	-.037060		10	.171415
			11	-.035397		11	.033978
			A	0.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
13	7	.22673	4	-.089854	.21095	4	-.184503
			5	.084779		5	-.237471
			6	.298868		6	-.205602
			7	.412411		7	.000000
			8	.298868		8	.205602
			9	.084779		9	.237471
			10	-.089854		10	.184503
			A	-.000000		C	-.000000
13	5	.23261	5	-.024512	.23849	5	-.524726
			6	.310761		6	-.229134
			7	.427501		7	.000000
			8	.310761		8	.229134
			9	-.024512		9	.524726
			A	.000000		C	.000000
13	3	.23276	6	.285408	.34462	6	-1.172345
			7	.429183		7	.000000
			8	.285408		8	1.172345
			A	-.000000		C	.000000
14	10	.19981	3	-.029130	.18722	3	-.025067
			4	-.040524		4	-.100183
			5	.031712		5	-.194902
			6	.195010		6	-.215877
			7	.342931		7	-.096717
			8	.342931		8	.096717
			9	.195010		9	.215877
			10	.031712		10	.194902
			11	-.040524		11	.100183
			12	-.029130		12	.025067
			A	-.000000		C	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
14	8	.20397	4	-.084293	.19029	4	-.138664
			5	.033235		5	-.197523
			6	.230005		6	-.218998
			7	.351053		7	-.098148
			8	.351053		8	.098148
			9	.230005		9	.218998
			10	.033235		10	.197523
			11	-.084293		11	.138664
			A	-.000000		D	-.000000
14	6	.20987	5	-.072811	.20711	5	-.402873
			6	.209606		6	-.275925
			7	.364205		7	-.105917
			8	.364205		8	.105917
			9	.209606		9	.235925
			10	-.072811		10	.402873
			A	.000000		D	-.000000
14	4	.21137	6	.130185	.25638	6	-.887192
			7	.369815		7	-.133033
			8	.369815		8	.133033
			9	.130185		9	.887192
			A	.000000		D	.000000
14	2	.21396	7	.500000	.49828	7	-1.988347
			8	.500000		8	1.988347
			A	0.000000		D	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
15	11	.18231	3	-.024130	.17150	3	-.018901
			4	-.039711		4	-.077336
			5	.002746		5	-.160423
			6	.123831		6	-.206228
			7	.269319		7	-.150096
			8	.335888		8	-.000000
			9	.269319		9	.150096
			10	.123831		10	.206228
			11	.002746		11	.160423
			12	-.039711		12	.077336
			13	-.024130		13	.018901
			A	.000000		C	.000000
15	9	.18528	4	-.076093	.17350	4	-.106028
			5	.003373		5	-.161909
			6	.126662		6	-.208321
			7	.274780		7	-.151670
			8	.342555		8	.000000
			9	.274780		9	.151670
			10	.126662		10	.208321
			11	.003373		11	.161909
			12	-.076093		12	.106028
			A	-.000000		C	.000000
15	7	.19065	5	-.094496	.18429	5	-.313609
			6	.172317		6	-.219540
			7	.284858		7	-.160124
			8	.354641		8	.000000
			9	.284858		9	.160124
			10	.172317		10	.219540
			11	-.094496		11	.313609
			A	.000000		C	-.000000
15	5	.19346	6	.026726	.22037	6	-.693911
			7	.291884		7	-.188073
			8	.362781		8	.000000
			9	.291884		9	.188073
			10	.026726		10	.693911
			A	-.000000		C	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
15	3	.19351	7	.319005	.33535	7	-1.404820
			8	.361990		8	-.000000
			9	.319005		9	1.404820
			A	-.000000		C	.000000
16	12	.16709	3	-.020142	.15819	3	-.014524
			4	-.037367		4	-.060473
			5	-.013593		5	-.131367
			6	.073662		6	-.186477
			7	.230627		7	-.172219
			8	.296514		8	-.070798
			9	.296514		9	.070798
			10	.230627		10	.172219
			11	.073662		11	.186477
			12	-.013593		12	.131367
			13	-.037367		13	.060473
			14	-.020142		14	.014524
			A	.000000		C	-.000000
16	10	.16967	4	-.067479	.15952	4	-.082363
			5	-.013403		5	-.132222
			6	.075214		6	-.187834
			7	.234147		7	-.173527
			8	.301521		8	-.071345
			9	.301521		9	.071345
			10	.234147		10	.173527
			11	.075214		11	.187834
			12	-.013403		12	.132222
			13	-.067479		13	.082363
			A	.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
16	8	.17440	5	-.101441	.16670	5	-.247285
			6	.078711		6	-.195068
			7	.211302		7	-.180517
			8	.311428		8	-.074253
			9	.311428		9	.074253
			10	.211302		10	.180517
			11	.078711		11	.195068
			12	-.101441		12	.247285
			A	.000000		C	-.000000
16	6	.17788	6	-.038060	.18991	6	-.549045
			7	.217916		7	-.282850
			8	.320144		8	-.083822
			9	.320144		9	.083822
			10	.217916		10	.282850
			11	-.038060		11	.548945
			A	.000000		C	0.000000
16	4	.17812	7	.178364	.25614	7	-1.071163
			8	.321636		8	-.110000
			9	.321636		9	.110000
			10	.178364		10	1.071163
			A	.000000		C	0.000000
16	2	.18145	8	.500000	.49454	8	-2.326576
			9	.500000		9	2.326576
			A	0.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEFF.	MSE	I	COEFF.
17	13	.15440	3	-.016948	.14677	3	-.011348
			4	-.033775		4	-.047878
			5	-.022361		5	-.107651
			6	.039373		6	-.163919
			7	.143717		7	-.175151
			8	.245754		8	-.115015
			9	.288481		9	.000000
			10	.245754		10	.115015
			11	.143717		11	.175151
			12	.039373		12	.163919
			13	-.022361		13	.107651
			14	-.033775		14	.047878
			15	-.016948		15	.011348
			A	.000000		D	.000000
17	11	.15646	4	-.059372	.14769	4	-.064903
			5	-.022374		5	-.108151
			6	.040189		6	-.164791
			7	.145925		7	-.176132
			8	.249322		8	-.115673
			9	.292620		9	-.000000
			10	.249322		10	.115673
			11	.145925		11	.176132
			12	.040189		12	.164791
			13	-.022374		13	.108151
			14	-.059372		14	.064903
			A	.000000		D	-.000000
17	9	.16555	5	-.100548	.15260	5	-.197332
			6	.042250		6	-.169417
			7	.150763		7	-.181354
			8	.256878		8	-.119172
			9	.301313		9	-.000000
			10	.256876		10	.119172
			11	.150763		11	.181354
			12	.042250		12	.169417
			13	-.100548		13	.197332
			A	-.000000		D	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
17	7	.16429	6	-.076144	.16819	6	-.440794
			7	.156165		7	-.197742
			8	.264815		8	-.130199
			9	.310728		9	.000000
			10	.264815		10	.130199
			11	.156165		11	.197742
			12	-.076144		12	.440794
			A	-.000000		C	-.000000
17	5	.16528	7	.074985	.20976	7	-.847435
			8	.268073		8	-.159293
			9	.313887		9	.000000
			10	.268073		10	.159293
			11	.074985		11	.847435
			A	.000000		C	.000000
17	3	.16587	8	.343482	.33068	8	-1.632487
			9	.313037		9	-.000000
			10	.343482		10	1.632487
			A	-.000000		C	.000000
18	14	.14349	3	-.014372	.13687	3	-.008597
			4	-.030399		4	-.038357
			5	-.026609		5	-.088544
			6	.016373		6	-.142010
			7	.099335		7	-.167494
			8	.195338		8	-.138745
			9	.260334		9	-.053884
			10	.260334		10	.053884
			11	.195338		11	.138745
			12	.099335		12	.167494
			13	.016373		13	.142010
			14	-.026609		14	.088544
			15	-.030399		15	.038357
			16	-.014372		16	.008597
			A	.000000		C	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	** LOCATION ** MSE	I	COEF.	** SCALE ** MSE	I	COEF.
18	12	.14515	4	-.052098	.13752	4	-.051812
			5	-.026708		5	-.088841
			6	.016772		6	-.142570
			7	.103699		7	-.168198
			8	.197798		8	-.178942
			9	.263545		9	-.054119
			10	.263545		10	.054119
			11	.197798		11	.178942
			12	.103699		12	.168198
			13	.016772		13	.142570
			14	-.026708		14	.088841
			15	-.052098		15	.051812
			A	.000000		D	-.000000
18	10	.14864	5	-.095707	.14097	5	-.159215
			6	.017922		6	-.145578
			7	.107856		7	-.171934
			8	.207302		8	-.142110
			9	.270627		9	-.055373
			10	.270627		10	.055373
			11	.207302		11	.142110
			12	.107856		12	.171934
			13	.017922		13	.145578
			14	-.095707		14	.159215
			A	.000000		D	.000000
18	8	.15235	6	-.096677	.15179	6	-.358677
			7	.107906		7	-.183524
			8	.209871		8	-.151984
			9	.278900		9	-.059233
			10	.278900		10	.059233
			11	.209871		11	.151984
			12	.107906		12	.183524
			13	-.096677		13	.358677
			A	.000000		D	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
18	6	.15406	7	.032392	.17941	7	-.685789
			8	.217878		8	-.176519
			9	.283730		9	-.069101
			10	.283730		10	.069101
			11	.217878		11	.176519
			12	.032392		12	.685789
			A	.000000		C	.000000
18	4	.15466	8	.216717	.25003	8	-1.249093
			9	.283283		9	-.094011
			10	.283283		10	.094011
			11	.216717		11	1.249093
			A	.000000		C	.000000
18	2	.15769	9	.500000	.49249	9	-2.660464
			10	.500000		10	2.660464
			A	0.000000		C	.000000
19	15	.17461	3	-.012278	.12821	3	-.007227
			4	-.027194		4	-.031059
			5	-.028188		5	-.073288
			6	.001206		6	-.122162
			7	.065843		7	-.154747
			8	.150590		8	-.147151
			9	.223692		9	-.093459
			10	.252657		10	.000000
			11	.223692		11	.093459
			12	.150590		12	.147151
			13	.065843		13	.154747
			14	.001206		14	.122162
			15	-.028188		15	.073288
			16	-.027194		16	.031059
			17	-.012278		17	.007227
			A	-.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	13	.13536	4	-.045722	.12868	4	-.041851
			5	-.028316		5	-.073387
			6	.001374		6	-.122525
			7	.065656		7	-.155242
			8	.152251		8	-.147637
			9	.226085		9	-.093762
			10	.255342		10	.000000
			11	.226085		11	.093762
			12	.152251		12	.147637
			13	.065656		13	.155242
			14	.001374		14	.122525
			15	-.028316		15	.073387
			16	-.045722		16	.041851
			A	.000000		C	.000000
19	11	.13872	5	-.089093	.13115	5	-.129770
			6	.001965		6	-.124442
			7	.068672		7	-.157865
			8	.156134		8	-.150217
			9	.231582		9	-.092368
			10	.261481		10	-.000000
			11	.231582		11	.092368
			12	.156134		12	.150217
			13	.068672		13	.157865
			14	.001965		14	.124442
			15	-.089093		15	.129770
			A	.000000		D	.000000
19	9	.14183	6	-.106726	.13886	6	-.294711
			7	.071541		7	-.165944
			8	.161279		8	-.158178
			9	.238611		9	-.097336
			10	.269268		10	.000000
			11	.238611		11	.097336
			12	.161279		12	.158178
			13	.071541		13	.165944
			14	-.106726		14	.294711
			A	.000000		C	0.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
19	7	.14403	7	-.046512	.15798	7	-.562861
			8	.165211		8	-.177759
			9	.243815		9	-.109575
			10	.274972		10	.000000
			11	.243815		11	.109575
			12	.165211		12	.177759
			13	-.046512		13	.562861
			A	.000000		C	-.000000
19	5	.14427	8	.116949	.20322	8	-.997319
			9	.244964		9	-.138291
			10	.276175		10	.000000
			11	.244964		11	.138291
			12	.116949		12	.997319
			A	.000000		C	0.000000
19	3	.14531	9	.362033	.32697	9	-1.856886
			10	.275934		10	-.000000
			11	.362033		11	1.856886
			A	0.000000		C	.000000
20	16	.12569	3	-.010562	.12057	3	-.005874
			4	-.024268		4	-.025417
			5	-.028202		5	-.060884
			6	-.038502		6	-.104810
			7	.041053		7	-.140091
			8	.113176		8	-.146714
			9	.185798		9	-.112568
			10	.231607		10	-.042343
			11	.231604		11	.042339
			12	.185900		12	.112574
			13	.113101		13	.146715
			14	.041049		14	.140099
			15	-.018602		15	.104810
			16	-.028262		16	.060884
			17	-.024268		17	.025407
			18	-.010562		18	.005874
			A	.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
20	14	.12679	4	-.040196	.12691	4	-.034167
			5	-.028333		5	-.069994
			6	-.008560		6	-.105047
			7	.041926		7	-.140436
			8	.114286		8	-.147189
			9	.187536		9	-.112861
			10	.233747		10	-.042454
			11	.233743		11	.042449
			12	.187538		12	.112867
			13	.114285		13	.147092
			14	.041522		14	.140474
			15	-.008560		15	.105547
			16	-.028333		16	.069904
			17	-.040196		17	.074167
			A	.000000		C	.000000
20	12	.12971	5	-.081887	.12272	5	-.106763
			6	-.008301		6	-.106207
			7	.042772		7	-.142263
			8	.116959		8	-.149089
			9	.191667		9	-.114417
			10	.238791		10	-.043048
			11	.238787		11	.043043
			12	.191668		12	.114423
			13	.116964		13	.149082
			14	.042768		14	.142261
			15	-.008301		15	.106297
			16	-.081887		16	.106763
			A	.000000		C	.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL SYMMETRIC CENSORING)

		** LOCATION **			** SCALE **		
N	M	MSE	I	COEF.	MSE	I	COEF.
20	10	.13254	6	-.108430	.12833	6	-.244336
			7	.044721		7	-.147875
			8	.120757		8	-.155197
			9	.197330		9	-.119219
			10	.245671		10	-.044859
			11	.245627		11	.044854
			12	.197372		12	.119214
			13	.120763		13	.155199
			14	.044717		14	.147873
			15	-.108479		15	.244336
			A	.000000		D	.000000
20	8	.13581	7	-.078068	.14199	7	-.467600
			8	.124255		8	-.169990
			9	.212274		9	-.130772
			10	.251479		10	-.049259
			11	.251475		11	.049263
			12	.212277		12	.130779
			13	.124261		13	.169992
			14	-.078012		14	.467623
			A	.000000		D	.000000
20	6	.13571	8	.041960	.17267	8	-.816575
			9	.204331		9	-.156576
			10	.253809		10	-.059004
			11	.253808		11	.058978
			12	.204328		12	.156621
			13	.041865		13	.816558
			A	.000001		D	.000001
20	4	.13584	9	.246572	.24624	9	-1.422961
			10	.253428		10	-.082329
			11	.253428		11	.082329
			12	.246572		12	1.422961
			A	.000000		D	.000000
20	2	.13951	10	.500000	.49175	10	-2.991259
			11	.500000		11	2.991259
			A	.000000		D	.000000

APPENDIX C

Computer Program

```
      PROGRAM COEFFS (INPUT, OUTPUT)
C      THIS PROGRAM COMPUTES AND TABLES THE COEFFICIENTS FOR BEST
C      LINEAR ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF
C      THE CAUCHY DISTRIBUTION FOR SAMPLE SIZES OF 5(1)20 WITH
C      ADDITIONAL CENSORING FROM ABOVE.  MUST READ IN THE EXPECTED
C      VALUES AND COVARIANCES OF THE ORDER STATISTICS FORMAT (6F12.6).
      DIMENSION EXP(20,20),COV(20,20),A(20,20),B(20),X(20),RL(20),
     1SE(20,20),AS(20,20),BS(20),XS(20),RKC(20),SES(20)
C      NN=MAXIMUM SAMPLE SIZE     MM=MINIMUM SAMPLE SIZE
      NN=20
      MM=5
      DO 5 N=MM,NN
      NI=N-2
      NR=N/2
      NM=(N+1)/2
      READ 101,(EXP(N,I),I=3,NM)
      DO 4 I=3,NR
    4 EXP(N,N+1-I)=EXP(N,I)
    5 CONTINUE
      DO 100 N=MM,NN
      NI=N-2
      DO 8 I=3,NI
    8 READ 101,(COV(I,J),J=I,NI)
      DO 10 I=3,NI
      DO 10 J=I,NI
   10 COV(J,I)=COV(I,J)
C      FILL THE A MATRIX
      A(1,1)=0.0
      A(1,2)=0.0
      A(2,1)=0.0
      A(2,2)=1.0
      DO 15 I=3,NI
      A(I,1)=1.0
   15 A(I,2)=-EXP(N,I)
      DO 16 J=3,NI
      A(I,J)=1.0
   16 A(2,J)=-EXP(N,J)
      DO 17 I=3,NI
      DO 17 J=3,NI
   17 A(I,J)=COV(I,J)+EXP(N,I)*EXP(N,J)
C      FILL THE B MATRIX
      B(1)=1.0
      DO 18 I=2,NI
   18 B(I)=0.0
C      COMPUTE THE COEFFICIENTS OR CENSOR AND COMPUTE THE COEFFICIENTS
      N4=N-4
      DO 30 ICEN=1,N4
      M=N-3-ICEN
      NP=M+2
      CALL MTXEL(NP,A,B,X)
      DO 20 I=3,NP
   20 RL(I)=X(I)
      RK=X(2)
      CALL SEL(N,M,NP,RK,RL,EXP,COV,SE)
```

```
C        FILL THE AS MATRIX
         DO 25 I=1,N4
         DO 25 J=1,N4
      25 AS(I,J)=COV(I+2,J+2)+EXP(N,I+2)*EXP(N,J+2)
C        FILL BS MATRIX
         DO 26 I=1,N4
      26 BS(I)=EXP(N,I+2)
C        COMPUTE THE COEFFICIENTS FOR THE SCALE PARAMETER
         NT=M
         CALL MTXEL(NT,AS,BS,XS)
         DO 27 I=1,NT
      27 RKC(I+2)=XS(I)
         CALL SEL(N,M,NP,1.0,RKC,EXP,COV,SES)
         SUMD=0.0
         DO 28 ID=3,NP
      28 SUMD=SUMD+RKC(ID)
         RD=SUMD
      30 CALL PRINT1(N,M,SE,RL,SES,RKC,RK,3,RD)
     100 CONTINUE
     101 FORMAT(6F12.6)
         STOP
         END
```

```
      SUBROUTINE PRINT1 (N,M,SEL,COFL,SES,COFS,A,I3,D)
      DIMENSION SEL(20,20),COFL(20),SES(20,20),COFS(20)
C      THIS ROUTINE PRINTS THE COEFFICIENTS AND MSE FOR SINGLE
C      CENSORING FROM ABOVE.
C      N=SAMPLE SIZE
C      M=SAMPLE SIZE AFTER CENSORING
C      SEL=MSE OF LOCATION ESTIMATOR
C      COFL= COEFFICIENTS FOR THE LOCATION ESTIMATE
C     SES  = MSE OF THE SCALE ESTIMATOR
C      COFS= COEFFICIENTS FOR SCALE ESTIMATE
C      A= CONSTANT FROM LOCATION ESTIMATE
C      D= CONSTANT FROM SCALE ESTIMATE
      IF(N.GT.5)GO TO 5
      IPAGE=39
      PRINT 20
      PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M),I3,COFS(3)
      M2=M+2
      IF(M.EQ.1)GO TO 4
      PRINT 22,(I,COFL(I),ICOFS(I),I=4,M2)
    4 PRINT 23,A,D
      K=37
      RETURN
    5 RM=M
      RK=K
      RLINE=RM+2.0
      IF(RLINE.GE.)GO TO 10
    8 PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M)I3,COFS(3)
      M2=M+2
      IF(M.EQ.1)go to 9
      PRINT 22,(I,COFL(I),I,COFS(I),I=4,M2)
    9 PRINT 23,A,D
      K=K-M2
      GO TO 16
   10 ISKIP=K+3
      DO 11 IS=1,ISKIP
   11 PRINT 24
      PRINT 25,IPAGE
      IPAGE= IPAGE+1
      PRINT 20
      K=41
   15 GO TO 8
   16 IF(N.EQ.20)GO TO 17
      RETURN
   17 IF(M.EQ.1)GO TO 18
      RETURN
   18 ISKIP=K+3
      DO 19 IS=1,ISKIP
   19 PRINT 24
      PRINT 25,IPAGE
      RETURN
```

```
   20 FORMAT(1H1,15X,13HGAM/MATH/72-3,///,41X,7HTABLE I,//,19X,51HCOEFFI
     1CIENTS FOR BEST CONDITIONAL ESTIMATION OF THE,/,17X,56HLOCATION AN
     2D SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION,/,25X,38H(WITH ADDIT
     3IONAL CENSORING FROM ABOVE),//,15X,59(*.*),/,28X,14H**LOCATION**
     4,16X,11H** SCALE **,/,16X,1HN,3X,1HM,5X,3HMSE,5X,1HI,4X,5HCOEF.,10
     5X,3HMSE,5X,1HI,4X,5HCOEF.,/,15X,59(*.*))
   21 FORMAT(1H0,14X,I2,I4,F10.5,I4,F11.6,F13.5,I4,F11.6)
   22 FORMAT(33X,I2,F11.6,15X,I2,F11.6)
   23 FORMAT(34X,1HA,F11.6,16X,1HD,F11.6)
   24 FORMAT(1H )
   25 FORMAT(42X,I3)
      END


      SUBROUTINE SEL(N,M,NP,RK,RL,EXP,COV,SE)
      DIMENSION RL(20),EXP(20,20),COV(20,20),SE(20,20)
C      THIS ROUTINE COMPUTES THE MSE FOR THE LOCATION AND SCALE
C      ESTIMATES
      SUM1=0.0
      SUM3=0.0
      DO 10 I=3,NP
      SUM2=0.0
      DO 5 J=3,NP
    5 SUM2=RL(I)*RL(J)*COV(I,J)+SUM2
      SUM1=SUM1+SUM2
   10 SUM3=RL(I)*EXP(N,I)+SUM3
      FA=SUM3-RK
      FA2=FA*FA
      SE(N,M)=SUM1+FA2
      RETURN
      END


      SUBROUTINE MTXEL(NP,A,B,X)
      DIMENSION A(20,20),B(20),X(20),PIV(20),C(38,38)
C      THIS ROUTINE IS A MODIFIED VERSION OF MTXEQ-MATRIX EQUATION
C      SOLVER SUBROUTINE, COMPUTER SCIENCE CENTER, WRIGHT-PATTERSON
C      AFB, OHIO
C      TO SOLVE THE LINEAR SYSTEM  AX=B
      DO 10 J=1,NP
      DO 10 I=1,NP
   10 C(I,J)=A(I,J)
      NPJ=NP+1
      DO 20 I=1,NP
   20 C(I,NPJ)=B(I)
      NP1=NP+1
      NPK=NP+1
      DO 120 I=1,NP
      IP1=I+1
      ATPE=0.0
      DO 40 J=I,NP
      IF (ABS(C(J,I))-ATPE) 40,30,30
   30 ATPE=ABS(C(J,I))
      IPIV=J
```

```
   40 CONTINUE
      IF (ATPE) 210,210,50
   50 DO 60 J=IP1,NPK
   60 PIV(J)=C(IPIV,J)/C(IPIV,I)
      IFROM=NP
      ITO=NP
   70 IF (IFROM-IPIV) 80,100,80
   80 RM=-C(IFROM,I)
      DO 90 J=IP1,NPK
   90 C(ITO,J)=C(IFROM,J)+RM*PIV(J)
      ITO=ITO-1
  100 IFROM=IFROM-1
      IF (IFROM-I) 110,70,70
  110 DO 120 J=IP1,NPK
  120 C(I,J)=PIV(J)
      I=NP
  130 IP1=I
      I=I-1
      IF (I) 160,160,140
  140 DO 150 J=NP1,NPK
      DO 150 L=IP1,NP
  150 C(I,J)=C(I,J)-C(I,L)*C(L,J)
      GO TO 130
  160 NPJ=NP+1
      DO 170 I=1,NP
  170 X(I)=C(I,NPJ)
  180 RETURN
  210 PRINT 1001
 1001 FORMAT(37HODET(A)=0 IN CALL TO SUBROUTINE MTXEL)
      RETURN
      END
```

```
      PROGRAM COEFFD(INPUT,OUTPUT)
C      THIS PROGRAM COMPUTES THE COEFFICIENTS FOR THE CONDITIONAL
C      BEST LINEAR INVARIANT ESTIMATION OF THE LOCATION AND SCALE
C      PARAMETERS OF THE CAUCHY DISTRIBUTION FOR SAMPLE SIZES 5(1)20
C      WITH ADDITIONAL SYMMETRIC CENSORING.  MUST READ IN THE EXPECTED
C      VALUES AND COVARIANCES OF THE ORDER STATISTICS, FORMAT(6F12.6)
C      NN=MAXIMUM SAMPLE SIZE      MM=MINIMUM SAMPLE SIZE
      NN=20
      MM=5
      DO 5 N=MM,NN
      NI=N-2
      NR=N/2
      NM=(N+1)/2
      READ 101,(EXP(N,I),I=3,NM)
      DO 4 I=3,NR
    4 EXP(N,N+1-I)=-EXP(N,I)
    5 CONTINUE
      DO 100 N=MM,NN
      NI=N-2
      DO 8 I=3,NI
    8 READ 101,(COV(I,J),J=I,NI)
      DO 10 I=3,NI
      DO 10 J=I,NI
   10 COV(J,I)=COV(I,J)
C      FILL THE A MATRIX
      A(1,1)=0.0
      A(1,2)=0.0
      A(2,1)=0.0
      A(2,2)=1.0
      DO 15 I=3,NI
      A(I,1)=1.0
   15 A(I,2)=-EXP(N,I)
      DO 16 J=3,NI
      A(1,J)=1.0
   16 A(2,J)=-EXP(N,J)
      DO 17 I=3,NI
      DO 17 J=3,NI
   17 A(I,J)=COV(I,J)+EXP(N,I)*EXP(N,J)
C      FILL THE B MATRIX
      B(1)=1.0
      DO 18 I=2,NI
   18 B(I)=0.0
C      COMPUTE THE COEFFICIENTS FOR THE BASIC CENSORED SAMPLE
      M=N-4
      NP=M+2
      CALL MTXEL(NP,A,B,X)
      DO 19 I=3,NP
   19 RL(I)=X(I)
      RK=X(2)
      CALL SELD(N,M,RK,RL,EXP,COV,SED)
C      FILL THE AS MATRIX
      N4=N-4
      DO 25 I=1,N4
      DO 25 J=1,N4
```

```
      DO 38 I=1,M
   38 RKC(I+2+ICEN)=XS(I)
      CALL SELD(N,M,1.0,RKC,EXP,COV,SES)
      IB=3+ICEN
      IT=IB+M-1
      SUMD=0.0
      DO 41 ID=IB,IT
   41 SUMD=SUMD+RKC(ID)
      RKD=SUMD
   50 CALL PRINT(N,M,SED,RL,SES,RKC,RK,IB,RKD)
  100 CONTINUE
  101 FORMAT(6F12.6)
      STOP
      END


      SUBROUTINE SELD(N,M,RK,RL,EXP,COV,SED)
      DIMENSION RL(20),EXP(20,20),COV(20,20),SED(20,20)
      SUM1=0.0
      SUM2=0.0
      NN1=(N-M)/2+1
      NNT=N-NN1+1
      DO 10 I=NN1,NNT
      DO 10 J=NN1,NNT
   10 SUM1=SUM1+RL(I)*RL(J)*COV(I,J)
      DO 12 I=NN1,NNT
   12 SUM2=RL(I)*EXP(N,I)+SUM2
      FNA=SUM2-RK
      FNA2=FNA*FNA
      SED(N,M)=SUM1+FNA2
      RETURN
      END


      SUBROUTINE PRINT(N,M,SEL,COFL,SES,COFS,A,I3,D)
      DIMENSION SEL(20,20),COFL(20),SES(20,20),COFS(20)
C      N=SAMPLE SIZE
C      M=SIZE AFTER CENSORING
C      SEL=MSE LOCATION
C      COFL=COEFFICIENTS FOR LOCATION ESTIMATE
C      SES=MSE SCALE
C      COFS=COEFFICIENTS FOR SCALE ESTIMATE
C      A=CONSTANT FROM LOCATION ESTIMATE
C      D=CONSTANT FROM SCALE ESTIMATE
      IF(N.GT.5) GO TO 5
      IPAGE=71
      PRINT 20
      PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M),I3,COFS(3)
      PRINT 23,A,D
      K=38
      RETURN
    5 RM=5
      RK=K
      RLINE=RM+2
      IF(RLINE.GE.RK)GO TO 10
```

```
   25 AS(I,J)=COV(I+2,J+2)+EXP(N,I+2)*EXP(N,J+2)
C        FILL THE BS MATRIX
      DO 26 I=1,M4
   26 BS(I)=EXP(N,I+2)
C        COMPUTE THE COEFFICIENTS
      NT=M
      CALL MTXEL(NT,AS,BS,XS)
      DO 27 I=1,NT
   27 RKC(I+2)=XS(I)
      CALL SELD(N,M,1.0,RKC,EXP,COV,SES)
      SUMD=0.0
      DO 40 ID=3,NP
   40 SUMD=SUMD+RKC(ID)
      RKD=SUMD
      CALL PRINT(N,M,SED,RL,SES,RKC,RK,3,RKD)
      IF(N.EQ.5) GO TO 100
      IF(N.EQ.6) GO TO 100
      IF(N.EQ.7) GO TO 100
C        CENSOR AND COMPUTE THE COEFFICIENTS
      GO TO 29
C        IF IT IS DESIRED TO CENSOR TO SIZE M=1, REMOVE PRECEEDING CARD
      IF((N/2)*2-N)28,29,28
   28 NC=(N/2)-2
      GO TO 30
   29 NC=N/2-3
   30 DO 50 ICEN=1,NC
      NPC=M+1
      NP=M+2
      M=M-2
      DO 31 JD=3,NPC
   31 A(2,JD)=A(2,JD+1)
      DO 32 ID=4,NP
      DO 32 JD=3,NPC
   32 A(ID,JD)=A(ID,JD+1)
      DO 33 ID=3.NPC
      DO 33 JD=2,NPC
   33 A(ID,JD)=A(ID+1,JD)
      NP=M+2
      CALL MTXEL(NP,A,B,X)
      DO 35 I=3,NP
   35 RL(I+ICEN)=X(I)
      RK=X(2)
      CALL SELD(N,M,RK,RL,EXP,COV,SED)
C        COMPUTE THE SCALE COEFFICIENTS FOR CENSORED SAMPLE
      NTS=M+1
      NT=M+2
      DO 36 IS=2,NT
      DO 36 JS=1,NTS
   36 AS(IS,JS)=AS(IS,JS+1)
      DO 37 IS=1,NTS
      DO 37 JS=1,NTS
      BS(IS)=BS(IS+1)
   37 AS(IS,JS)=AS(IS+1,JS)
      CALL MTXEL(M,AS,BS,XS)
```

```
8 PRINT 21,N,M,SEL(N,M),I3,COFL(I3),SES(N,M),I3,COFS(I3)
  M2=I3+M-1
  M4=I3+1
  IF(M.EQ.1)GO TO 9
  PRINT 22,(I,COFL(I),I,COFS(I),I=M4,M2)
9 PRINT 23,A,D
  K=K-M-2
  GO TO 16
10 ISKIP=K+3
   DO 11 IS=1,ISKIP
11 PRINT 24
   PRINT 25,IPAGE
   IPAGE=IPAGE+1
   PRINT 20
   K=41
15 GO TO 8
16 IF(N.EQ.20)GO TO 17
   RETURN
17 IF(M.EQ.1)GO TO 18
   RETURN
18 ISKIP=K+3
   DO 19 IS=1,ISKIP
19 PRINT 24
   PRINT 25,IPAGE
   RETURN
20 FORMAT(1H1,15X,13HGAM/MATH/72-3,///,41X,8HTABLE II,//,19X,51HCOEFF
  1ICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE,/,17X,56HLOCATION A
  2ND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION,/,26X,37H(WITH ADDI
  3TIONAL SYMMETRIC CENSORING),//,15X,59(*.*),/,23X,14H** LOCATION **
  416X,11H** SCALE **,/,16X,1HN,3X,1HM,5X,3HMSE,5X,1HI,4X,5HCOEF.,10
  5X,3HMSE,5X,1HI,4X,5HCOEF.,/,15X,59(*.*))
21 FORMAT(1H0,14X,I2,I4,F10.5,I4,F11.6,F13.5,I4,F11.6)
22 FORMAT(33X,I2,F11.6,15X,I2,F11.6)
23 FORMAT(34X,1HA,F11.6,16X,1HD,F11.6)
24 FORMAT(1H )
25 FORMAT(42X,I3)
   END
```

Vita

Ralph Merle Spory, Jr. was born 22 April 1940 at New Florence, Pennsylvania, the son of Ralph M. Spory and Mollie I. Spory. After graduating in 1958 from Laurel Valley Joint High School, Bolivar, Pennsylvania, he entered the United States Air Force Academy. He graduated from the Air Force Academy in 1962 with a Bachelor of Science degree and a commission as Second Lieutenant in the United States Air Force. In 1963 he graduated from Pilot Training and spent the next seven years in various flying assignments. He entered the Air Force Institute of Technology in June 1970. He is married to the former Karen Ann Benson of New York City, New York.

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This thesis was typed by Mrs. Anna L. Lloyd.